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SURFACE TEMPERATURE THERMAL FLUXMETERS FOR SHOCK TUBES

by

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Summary

This article reviews the principle of the surface temperature thermal fluxmeter that is generally used for measurements in shock tubes. Pickups of this type are described; they were built and experimented upon at the ONERA and enable heat flux measurements either on the wall or at the impact point of hemispherical models. The calibration of these fluxmeters raises a delicate problem and the processes used are dealt with in an in-depth study.

Key words (NASA thesaurus): Heat flux - Measurement - Heat transfer - Hypersonic heat transfer - Gas - Solid interfaces - Shock tubes - Measuring instruments - Instrument transmitters - Transducers - Calibrating - Surface temperature - Wall temperature.

NOTATION

T	temperature rise (index S for the surface temperature)
t	time
ϕ	density of heat flux (index 0 for a constant flux)
λ	thermal conductivity
ρ	density
C	specific heat
a	diffusivity $\left(a = \frac{\lambda}{\rho c}\right)$

P thermal potential $\left(P = \int_0^T \lambda dT\right)$

ε thickness

σ electrical conductivity } of the thermal

R electrical resistance } element

α temperature coefficient }

I amperage

V potential drop

r curve radius of the heat sensitive element

2

τ integration time variable

p operational variable

$\theta(p)$ Laplace temperature transform $\left(\theta = \int_0^\infty e^{-pt} \cdot T dt\right)$

$\Phi(p)$ Laplace flux density transform $\left(\Phi = \int_0^\infty e^{-pt} \cdot \varphi dt\right)$

erf(x) error function $\left(erf[x] = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} \cdot du\right)$

erfc(x) complementary error function (erfc[x]=1-erf[x])

$E_i(-x)$ integral exponential $\left(E_i[-x] = \int_\infty^x \frac{e^{-u}}{u} du\right)$

$I_0(x)$ Bessel imaginary function of order zero and of the first species

$K_0(x)$ Bessel imaginary function of order zero and of the second species

$K_1(x)$ Bessel imaginary function of first order and of the second species

I. INTRODUCTION

The surface temperature fluxmeters is a device which, thanks to its short response time, has been used extensively for the past 15 years for measurements in flows of short duration, particularly in shock tubes. This type of fluxmeter is generally designed in publicatıcns by the name of thin layer thermometer, which is not entirely correct since the thermometer is only one of the elements of the pickup. The heat flux to be measured is absorbed at the surface of a material whose thickness is sufficient so that the temperature rise of the unexposed side is negligible. The variations in surface temperature may b e linked to the value of the incident flux by a theoretical law involving the thermal properties of the material. The correlation established between the flux and the surface temperature does not postulate any delay in the response of the pickup, at least when the continuous heat theory is valid. The response time is only due to the thermal inertia of the surface thermometer which we seek to minimize as much as possible.

This notion of response time is not always self-evident for the users. What's more, the usual calibration processes lack precision for reasons which have not yet been clearly revealed. This is why a complete study of the pickup, both from a theoretical and experimental standpoint, was undertaken; this study namely resulted in the construction of a certain number of fluxmeters which were used in the pilot shock tube of the ONERA in Chalais-Meudon. The description of this shock tube in addition to the results of the first thermal measurements are dealt with in reference 1.

II. PRINCIPLE

The principle of the device is based on the law of heat diffusion in a semi-unlimited medium. A relatively good thermal

insulating material absorbs the convection flux generated by the flow; the signal is diffused in the matter according to a direction perpendicular to the exposed surface and does not reach the other end of the fluxmeter throughout the duration of the phenomenon. On account of the few hundreds of microseconds that the flow lasts, this imposes a pickup thickness that must not be exceeded and which is determined through computation. The surface temperature variation of the insulating material which we shall call the heat sensitive element, is subtracted from the flux absorbed based on the Fourier theory. The property of the heat sensitive element which comes into play in the formula is expressed by the product of the conductivity times the density and the specific ehat; this value is sometimes referred to by the name of effusivity or thermal chunking coefficient.

The surface temperature is measured with a metal resistor thermometer deposited in the form of a very thin layer whose thickness controls the pickup's response time.

It is of interest to place this type of fluxmeter among the devices used for the heat transfer measurements. Table I shows a collection of the characteristics of different models of heat flux pickups.

TABLE I

Nature des échanges de chaleur (1)	Régime thermique (2)	Type de fluxmètre (3)	Propriétés de l'élément calorimétrique utilisées (4)	Grandeurs mesurées (5)
Rayonnement (6)	—	Pyromètre optique (7)	—	Énergie électromagnétique (8)
(9) Conduction Convection Rayonnement (après transformation en chaleur)	(10) Régime permanent	A circulation de fluide A changement d'état (11) A refroidissement par rayonnement A gradient thermique A chauffage électrique	Chaleur massique Chaleur de transformation (12) Facteur l'émission Coductibilité thermique —	débit, échauffement débit de matière subissant le changement d'état (13) mpérature d'équilibre gradient de température puissance électrique dissipée
	Régime variable (14)	(15) A inertie thermique A température superficielle	Masse volumique Chaleur massique (16) masse volumique Chaleur massique Conductibilité thermique	dérivée de la température par rapport au temps dérivées de la température dans plusieurs tranches d'épaisseur variations de la température de surface (17)

Key to table I

1 - Types of heat exchanges

2 - Thermal condition

3 - Type of fluxmeter

4 - Properties of heat sensitive elements used

5 - Parameters measured

6 - Radiation

7 - Radiation pyrometer

8 - Electromagnetic energy

9 - Conduction, convention, radiation (after heat transfer)

10 - Permanent operation

11 - Fluid circulation

change in state

thermal gradient

electrical heating

12 - Specific heat

transformation heat

emission factor

thermal conductivity

13 - flow, heat rise

flow of matter undergoing change in state

equilibrium temperature

temperature gradient

electrical power dissipated

14 - Variable operation

15 - Thermal inertia

surface temperature

16 - Density, specific heat

Density, specific heat, thermal conductivity

17 - Derivative of the temperature with respect to the time

Derivatives of the temperature in several thickness ranges

Variations in the surface temperature

These are classified in two very different categories; the first includes thermal radiation measuring devices in which the essential element is a specific detector of electromagnetic energy such as a photoelectric cell.

The second category of pickups is intended for measuring conduction or convection thermal fluxes as well as radiant fluxes provided that the radiation is transformed into heat in a suitable absorbing medium such as carbon black. These pickups are divided into two groups: the first contains continuous operation fluxmeters in which the temperature is independent of the time when the device is subjected to a constant flux; the second group contains variable operation fluxmeters whose temperature changes with time when exposed to a constant flux.

In the continuous operation devices the flux absorbed is balanced by an equal flux created in the pickup and which lends itself to the measurement. This flux can be generated by the circulation of a fluid such as water or by changing the physical state of a consumable substance. One can also use the pickup radiation at a given temperature, or a temperature gradient in a material of known conductivity. Another traditional method consists in heating the fluxmeter with an electric current, with the dissipated power being equal to the flux to be measured when equilibrium is reached; it would also be possible to compensate for the relatively low flux with cooling by means of the Peltier effect.

The variable operation fluxmeters firstly include the thermal inertia pickups in which the derivative with respect to time of the temperature of an isothermal calorimetric element is measured. If this isothermal condition is not fulfilled, for example due to excessively large dimensions of the pickup, it is possible to divide the pickup into several separately isothermal elements and to add the temperature derivatives of each element.

This avoids involvement of the thermal conductivity. However, if we take this value into account, we can theoretically deduct the flux absorbed by a calorimetric element based on the change in a local temperature; in this way it is easier to use the surface temperature of a medium of almost infinite thickness on account of the duration of the measurement. We thus arrive at the principle of a surface temperature fluxmeter.

These different methods of heat flux measurement are explained in greater detail in the reference works (2).

III. THEORY

The theoretical principles behind the fluxmeter are traditional and have been dealt with in numerous publications; among them, one of the best documented ones is the note by R. Vidal which marked the beginning of the generalized use of surface temperature pickups (3). We shall first review the elementary theory of the flat surface fluxmeter having constant thermal properties; the relations between the heat flux and the surface temperature which result are used to calculate the thermal transfers to the shock tube. We shall then examine the causes for the differences between the actual response of a pickup and that which is deduced from the elementary principles discussed earlier. These causes reside in the curvature of the surface (if applicable), in the thermal inertia of the surface thermometer, and in the variations in the properties of the calorimetric element with temperature changes.

III.1 FLAT PICKUP WITH CONSTANT THERMAL PROPERTIES

If x represents the distance to the surface of a point on the calorimetric element, the heat equation is written as follows:

(1) $$a\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

Assuming that T is the temperature rise value which is zero at the beginning time, this equation, through Laplace transforms, becomes:

(2) $$a\frac{\partial^2 \theta}{\partial x^2} = p\theta.$$

We set a zero temperature rise for infinite x as the limit conditions, which translates into the semi-unlimited nature of the medium where the heat is dissipated and a surface thermal gradient (x = 0) of the value: $-\frac{\varphi}{\lambda}$.

Taking into account these conditions, if Φ designates the transform of the flux density φ, the solution for x = 0 of equation 2 is expressed as:

(3) $$\theta_s = \frac{1}{\sqrt{\lambda\rho c}} \cdot \frac{\Phi}{\sqrt{p}}.$$

The value of the surface temperature rise Ts, function of φ is deducted from 3, and inversely the expression of φ function of Ts. That is:

(4) $$T_s = \frac{1}{\sqrt{\pi\lambda\rho c}} \int_0^t \varphi(\tau) \frac{d\tau}{\sqrt{t-\tau}}$$

(5) $$\varphi = \sqrt{\frac{\lambda\rho c}{\pi}} \int_0^t \frac{dT_s}{d\tau} \cdot \frac{d\tau}{\sqrt{t-\tau}}$$

Starting with the surface temperature recording, calculation of φ with equation 5 is performed using a digital machine. The usual method applied is the trapezium method which is used after integrating by parts in order to avoid calculating the derivative $\frac{dT_s}{d\tau}$ which is always unprecise. The contribution of the last integral element is undetermined because, for τ close to t, it contains the quotient of two quantities close to zero. Therefore, in order to remove the imprecision, we are forced to hypothesize on the function $\phi(\tau)$, the simplest hypothesis being to assume a constant value for the flux density during the last integration interval.

This approximate calculation process results in errors which are linked to the shape of the curve Ts (τ). It is shown that in certain cases the increase in the number of integration steps can arrive at an improvement in the error, and that greater precision is obtained if the integral of 5 is calculated based on a limited development of Ts in powers of $\sqrt{\tau}$. The optimum flux calculation method thus depends on the shape of the experimental curve.

Another important question is raised concerning the minimal thickness to be given to the calorimetric element in order for

the element to fulfill the conditions of a semi-unlimited medium. For example, we can assume that at instant t corresponding to the end of the measurement, the thermal signal which affects the end of the pickup must not exceed 1% of its value on the surface exposed to the flux. This thermal signal can be defined either by the temperature rise or by the heat flux density

4

which goes through an isothermal plane. To get an idea of the minimal distance from the surface where the foregoing conditions are fulfilled, we can perform the calculation while assuming that the pickup is subjected to a constant flux of density φ_0 established in the form of an increment. According to equation 2 and considering the conditions at the limits, the temperature rise at an abscissa point x has the following transform:

(6) $$\theta(x) = \frac{\varphi_0}{\sqrt{\lambda \rho c}} \cdot \frac{e^{-\sqrt{\frac{p}{a}}x}}{p^{3/2}}.$$

From this equation we deduce the value of the temperature rise:

(7) $$T(x) = \frac{\varphi_0}{\sqrt{\lambda \rho c}} \left[\frac{2\sqrt{t}}{\sqrt{\pi}} \cdot e^{-\frac{x^2}{4at}} - \frac{x}{\sqrt{a}} \operatorname{erfc} \sqrt{\frac{x^2}{4at}} \right].$$

The flux density at x is given by:

(8) $$\varphi(x) = -\lambda \frac{\partial T}{\partial x} = \varphi_0 \cdot \operatorname{erfc} \sqrt{\frac{x^2}{4at}}.$$

From equations 7 and 8 we show that the ratios of $\frac{T(x)}{T_0}$ and $\frac{\varphi(x)}{\varphi_0}$ reach only 1% for $x = 3.16\sqrt{at}$ and $x = 3.74\sqrt{at}$. We can therefore assume that the semi-unlimited medium condition is correctly satisfied provided that the thickness is at least equal to $4\sqrt{at}$. This corresponds to about 0.1 mm for a calorimetric element made of non-crystalline silica or pyrex subjected to a flow of 600 micro-s. For reasons of mechanical strength the thickness of the calorimetric elements is very much in excess of the minimum necessary; we shall see that this enables the pickups to be subjected to heat fluxes of much greater duration during the calibrations.

III.2 INFLUENCE OF THE SURFACE CURVATURE

The equations which have just be established are only valid if the sensitive surface of the fluxmeter is flat. However, thermal measurements in shock tubes must generally be made on curved surfaces; this is how the wall fluxes are observed inside a cylindrical tube and the impact fluxes are generally measured on spherical nose models. The heat equation takes on the differential form of 1 and calculation of the thermal fluxes using 5 may be fraught with a certain degree of error; intuitively, this error is negligible if the penetration depth of the thermal signal, in the vicinity of $4\sqrt{at}$, is small compared to the curve radius r of the pickup surface.

The calculation, which is dealt with in Appendix 1 for concave cylindrical and convex spherical surfaces, allows us to set the validity limits of the planar approximation. In this way, for a spherical shaped silica calorimetric element, the surface temperature differs at the end of one millisecond by less than 1% from that of a flat-surfaced element, if the radius reaches 3 millimeters.

Under the experimental conditions encountered during the shock tube tests at the ONERA, the influence of the curvature is entirely negligible. However, the same does not apply to the calibrations performed over somewhat longer durations.

III.3 RESPONSE TIME

If the surface temperature could be measured without any delay, the pickup would be capable of instantaneously responding to the incident heat flux. The thermometer used is a variable metal resistance deposited in the form of a thin layer whose thickness does not exceed 0.1 micron. Although very small, this thickness implies a certain amount of inertia for the heat sensing element, which results in the response time.

This response time can be defined in many ways, with each definition leading to a different digital value. The primary thermal measurements in the shock tubes concern either the wall flux in a state of streamline flow or the impact point fluxes. In the first instance, the thermal flux is more or less proportional to $\frac{1}{\sqrt{t}}$, which, according to 4. causes a constant surface temperature rise. However, the impact fluxes are somewhat constant, whence a surface heating proportional to the square root of the time.

The following two definitions of the response time are thus adopted:

a) The time necessary for the measured temperature rise to reach a given fraction of that which corresponds to equilibrium with a flux proportional to $1/\sqrt{t}$;

b) The time necessary for the flux density, calculated according to the equation that is valid for a constant flux and based on the measured temperature rise, to reach a given fraction of the constant flux density actually applied.

In any event the measured temperature is the average temperature of the thin layer throughout its thickness ε which is assumed to be uniform to simplify the calculation.

It is reasonable to choose a deviation from equilibrium equal to 1% or 5% in order to characterize the response time. We must emphasize the fact that a value of 1/2.718 for this deviation has absolutely no useful meaning; the corresponding response time, generally called the time constant, can only be considered for a measuring device whose response takes on an exponential form. But the response of a surface temperature fluxmeter obeys a totally different law.

From the results of Appendix 2, in which the expanded calculation is given, the 5% response time with a constant flux is equal to 128 times the response time at 1/2.718; however, for an exponential phenomenon as a function of time, the ratio of these two response times is close to 3. An abusive extension of the notion of a time constant to the thermal fluxmeter therefore runs the risk of introducing a considerable error in the evaluation of the real response time, since a deviation of 1/2.718 translates into about 37%.

A theoretical examination of the problem reveals a characteristic time of the pickup which is expressed by the formula:

$$t_c = \frac{\rho' c' \varepsilon^2}{\lambda \rho c} \qquad (9)$$

In this equation ρ', c' and ε represent the density, specific heat, and the thickness of the thin heat-sensitive layer, respectively, while $\lambda\rho c$ is the thermal removal coefficient of the calorimetric element. In the case of a fluxmeter made of

non-crystalline silica having a platinum thermometer 0.1 micron thick, the characteristic time is 3 x 10-8 second.

The different response times t_R are proportional

5

to the characteristic time. The results of the calculations can be put into the following form:

	t_R/tc (5 %)	t_R/tc (1 %)
1 Échelon de temperature	134	3.181
2 Échelon de flux	280	7.850

Key to table

1 - Temperature increment

2 - Flux increment

By adopting the above-mentioned numerical value for t_c, the response time within 5% under a constant flux reaches 8.4 micros but it takes 28 times longer to reduce the deviation to 1%. Just as in 9, the response time is proportional to the square of the thin layer thickness; we can see that it would be delicate to take measurement in a flow lasting a few hundred microseconds with heat sensitive elements greatly in excess of 0.1 micron.

III.4 INFLUENCE OF VARIATIONS IN THERMAL PROPERTIES WITH TEMPERATURE

Equation 1 only has an exact analytical solution if the diffusivity is independent of the temperature. This is not absolutely true, since the thermal properties λ, ρ and c always undergo variations to a greater or lesser degree. Naturally, if

the temperature rise of the pickup remains low, these variations do not give rise to substantial errors. But we should determine theoretically the amplitude of these errors.

The calculation, which is dealt with in Appendix 3, is inspired after a method explained in reference work 4 in which some imprecisions were corrected following the observations made in reference 5. This calculation, which is only approximative, uses a disturbance method. The heat equation is strictly written as:

(10) $$\frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) = \rho c \frac{\partial T}{\partial t}$$

where λ, ρ and c are functions of T. From the thermal potential $P = \int_0^T \lambda dT$, expression 10 becomes:

(11) $$a(P) \cdot \frac{\partial^2 P}{\partial x^2} = \frac{\partial P}{\partial t}$$

P is calculated as the sum of two terms: P_0, the solution to an equation similar to 11 in which we let a(P) = a_0 (constant corresponding to the initial temperature) and $P_1 \ll P_0$, obtained from 11 where we write: P = P0 + P1 and a(P) = $a_0 + a_1$, with $a_1 \leq a_0$ and a function of P. The expression of the surface temperature remains to be determined from P by obtaining a law of variation of λ with T.

In the case of a material such as silica, it is difficult to deduce the behavior of the variations a_1 (P) and λ (T) from the data of tables. In this regard the various experimenters presented contradictory results which only agreed on a tendency of the thermal conductivity to increase with rising temperatures. For lack of more precise information, we chose the values

proposed by reference 4 in the digital applications for the ambient temperature and 50 degrees C, assuming linear variations in a and λ during this interval.

We then calculate a value of the surface heating under a constant flux 1.7% less than that which would be obtained from equation 1 if the heating is close to 30 degrees C.

Several authors imagined analog circuits which could replace the digital methods of calculating the flux based on 5 [6]. It is therefore conceivable, thanks to a careful choice of variable elements for such circuits, to take into account the modifications in the thermal properties with changes in temperature [5].

IV. PICKUP ELEMENTS

A surface temperature thermal fluxmeter has a calorimetric element which is the essential component of the pickup, and a thermometer element usually consisiting of a variable thin layer resistor. We shall present the materials chosen for these elements as well as the fabrication processes. We shall furthermore examine the different ways of making the electrical terminals which make the connection between the ends of the thermometer element and the external electrodes of the pickups.

IV.1 CALORIMETRIC ELEMENT

The surface heating, which provides the signal coming from the pickup, increases as the thermal conductivity decreases, as indicated by equation 4. It is therefore important to choose a fairly well-insulating material having thermal properties which are as reproducible as possible when going from one sample to another. Non-crystalline transparent silica properly satisfies these conditions; if an x-ray inspection does not reveal any traces of crystallization, the thermal conductivity can be

considered isotropic. Several users preferred pyrex; this material, however, seems less advantageous since its chemical makeup is not always consistent.

The product ρc of the density times the specific is relatively easy to determine and quite constant for different bars of silica. THe values presented in the tables range from 1.60 to 1.70 J/cubic centimeter degree C at ambient temperature. Some measurements made at the CNRS laboratory in Marseille yielded values falling within this range. For example, by using 1.66/ J/cm3degreeC, the error must be less than plus or minus 3%.

The thermal conductivity λ poses a much more delicate problem. This vlue is particularly sensitive to the presence of impurities and, in addition, is very difficult to measure accurately. As we have seen, it is impossible to obtain sufficiently sure information in the available literature concerning the variations in λ with temperature. But even at 20 degrees C the values given in the tables show considerable deviations, with some being twice as great as others.

R. Missenard simply indicates a thermal conductivity between 13.9 and 18.6 mW/cm degree C [4]. However, a more recent publication [8] proposes a value of 14.3 mW/cm degree C at ambient temperature, which can be reproduced accurately for varied samples of material; according to the author, the spread observed in the tables are only due to measurement errors. Without being quite to optimistic, the conclusions which were drawn from the calibrations performed on the fluxmeters allow us to hope for a relative consistency in the thermal properties from one silica bar to another, to within a few centimeters.

The silica machining processes are quite easy and, what's more, it is possible to achieve excellent surface conditions

from the point of view of aerodynamics. In addition, the shape lends itself well to the depositing of thin layers using conventional methods; this is therefore a well-suited material for thermal measurements in shock tubes.

IV.2 THERMOMETER ELEMENT

The thin layer themometer made for measuring the surface temperature involves several depositing techniques which have been tested at the ONERA.

IV.2.1 THIN LAYERS APPLIED BY PAINTING

Several foreign suppliers have perfected a paint whose exact composition has not been revealed, but which contains a liquid organic binder containing very fine metal particles in suspension. This paint is applied with a brush to the surface of the optically polished and very carefully degreased silica. A baking in the vicinity of 700 degrees C in a well ventilated oven ensures pyrolysis of the binder and a chemical etching of the surface of the silica by means of a halogen contained in the paint. The metal particles are thus deposited in a highly adhesive layer whose thickness is close to 0.1 micron, according to the suppliers.

The metal used for the thermometric resistors is platinum. The reproducibility of the resistance values from one deposit to another can only be obtained by trial and error, and it is difficult to ensure it within a range of less than plus or minus 10%. Nevertheless, after baking one can increase the value of the resistor by delicate erosion with an abrasive material.

The temperature coefficient is defined by the expression:

(12) $$\alpha = \frac{1}{R_0} \cdot \frac{dR}{dT}$$

where r_0 designates the values of the resistor at 0 degrees C. This temperature coefficient is always noticeably lower than that of the pure metal; in addition, it varies from one thin layer to another and depends on the shape of the silica surface. With the paint used, α is equal to 15 x 10-4 (degrees C)-1 plus or minus 20% if the surface is concave cylindrical, 25x 10-4 (degrees C)-1 plus or minus 10% if this surface is flat, and it ranges between these two values for convex spherical surfaces. Pure platinum has a temperature coefficient equal to 39.2 x 10-4(degrees C)-1.

The difference between the coefficients of pure platinum and of the deposits made seems difficult to explain by the well-known thin layer effect which occurs only for much smaller metal thicknesses. Most authors prefer to assume the action of impurities in the platinum [9]. This change in the temperature coefficient would not be a problem for the measurement if its value remained stable with time. However, in the case of thin layers deposited by painting, we are often witness to spontaneous changes both in R0 and α . Attempts at stabilization by annealing did not yield the desired results. The changes in the resistance are of a random nature and two layers seemingly made under the same conditions can have entirely different behaviors. In certain cases the resistance varies, at constant temperature, by a few thousandths in the space of about one hour: such instability makes the thermometer element unusable. Usually the resistance tends to increase, with the variation being a few hundredths over a few months. As for the temperature coefficient, it can change in either direction. Sometimes

certain thin layers are perfectly stable, with the variations observed in the course of a year being less than the measurement error.

This random nature of the stability of the thin layers means a need for frequent calibration. When the pickups are used for impact point flux measurements, the dust contained in the flow causes erosion, too, which noticeably increases the value of the resistance, whence the need to measure it before and after each test. On the other hand, the flux measurements on the wall of the shock tube are not affected in this way.

Microscopic inspection of the thin layers reveals a heterogeneous structure and, in particular, considerable local variations in thickness. This is why another method of depositing was tested in the hopes of obtaining both even and stable layers.

IV.2.2 THIN LAYERS DEPOSITED BY CATHODE SPUTTERING

The process of cathode sputtering allows us to obtain resistors whose values are more reproducible than with paints. The equipment consists of a cathode made of the metal to be deposited raised to a potential of 2,500 volts with respect to an anode connected to ground. A sample mounting block which holds the silica base is first heated to a temperature of 300 degrees C and is at the anode potential. The whole setup is placed in an argon atmosphere at a pressure of 0.1 millibar.

The surface of the silica must be carefully cleaned; it is covered with a mask which limits the deposit, which is generally made in the shape of a rectangle having a length of about 5 mm and a width one tenth that size.

To obtain a resistance of 100 ohms the duration of exposure is close to one minute. The reproducibility of the deposit time

usually provides for relatively accurate reproducibility of the resistance value. The structure of the layers obtained by this process, as revealed by an inspection at very high magnification, seems more regular than that of the painted-on layers. Still, several heterogeneities remain and must result in local variations in thickness.

We attempted to make an approximate measurement of the average value of this thickness using an interferometry method. The silica base with its thin layer is covered with with another reflecting platinum layer deposited using the same process and which creates an additional thickness close to that of the layer to be measured. At a small distance above the base a semi-reflective blade is placed slightly inclined so as to form an air corner. The blade is illuminated with monochrome light and two fringe systems are observed: one corresponds to the portion of the reflecting platinum layer covering the thin layer to be measured, the other the portion of this reflecting layer deposited directly on the silica. The difference in level, which is equal to the thickness to be determined, is measured by the spread between the two fringe systems. The values obtained for thin layers of about one hundred ohms resistance. is situated in the vicinity of 0.5 micron.

In the case of thermometer elements obtained by cathode sputtering, two metals were used: platinum and nickel. The platinum deposits made directly on the silica greatly lack mechanical strength. Adherence is considerably improved by first putting an extremely thin underlayer made of either chromium or nickel. As for pure nickel deposits, they are applied directly on the silica. However, regardless of the metal used, the bonding of the thin layers applied by cathode sputtering is markedly poorer

7

than with paint. Tests conducted in a shock tube nevertheless have shown that the adherence was sufficient for wall measurements. But paint remains the only valid way for impact point fluxmeters in which the dust causes very strong removal forces.

Platinum thin layers deposited by cathode sputtering have excellent stability, with the R_0 and α values remaining practically constant over time. This is not true with nickel layers which generally become unstable after the calibrations under a radiating flux which will be described later on.

The α temperature coefficients of the platinum layers depend mainly on the resistance values. They vary approximately from 15 to 20 x 10-4(degrees C)-1 for resistances of 50 ohms, to less than 10 x 10-4(degrees C)-1 for 300 ohms. The thin layers made of nickel have a totally random temperature coefficient ranging between 10 and 45 x 10-4(degrees C)-1 whereas, in the case of the pure metal, the value of α is 66.3 x 10-4 (degrees C)-1.

IV.3 ELECTRICAL TERMINALS

The electrical connecting of the thin layers to the output wires of the fluxmeters can be made by two methods.

a) The installation of solid wires crossing through the calorimetric element and fastened with a cement to the silica (figure 1a);

b) The depositing of thin layers having a negligible resistance and covering the calorimetric element up to the points of junction with the output wires (figure 1b).

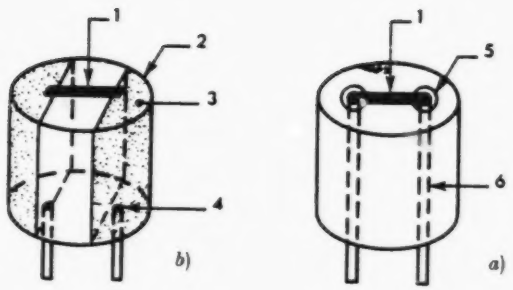

Figure 1 - Placing of Terminals

1 - Thermometer element

2 - Rounded edge

3 - Thin silver layer

4 - Wire soldering

5 - Deposit of silver in a thin layer

6 - Sealed silver wire

The first method was tested using two silver wires of 0.6 millimeters in diameter sealed in holes drilled in the silica. The experiment showed that the surface connection of the thin layer to these solid terminals raised a delicate problem; the surface condition in the vicinity of the junctions is not satisfactory and the electrical contact turns out to be quite unreliable. In addition, the terminals constitute heat sinks which disturb the shape of the isothermal surfaces in the calorimetric element resulting in a non-negligible error in the measurement of the surface temperature.

This is why the second process was finally adopted. The thin layers making up the terminals are made from a silver paint which is applied in a similar fashion to that of the platinum paint. The electrical resistance of these layers is perfectly negligible and the contact with the thermometer film is very good, provided that this film overlaps considerably on the silver terminals. Since the baking temperature of the silver paint is 500 degrees C, the terminals are made after applying the thermometer element if this element is applied with a paint, and before if the platinum film is made from cathode sputtering.

The fastening of copper wires to the thin silver layers does not run into any difficulties provided that a tin solder having a low melting point is used.

V. FLUXMETERS MADE

The thermal pickups made for the pilot shock tube include wall fluxmeters, hemispherical nose models for impact point measurements, and crosswise shock wave detectors capable of providing a signal when the flow travels through the axis of the tube.

V.1 WALL FLUXMETERS

These fluxmeters are intended for taking measurements at the wall of a shock tube having a curvature radius of 30 mm. The model used, shown in figure 2, is assigned number "20 L 101 a." I has a silica calorimetric element (1) whose sensitive surface conforms to the cylindrical shape of the tube. The platinum thermometer element (2) has a rectangular shape of about 5 mm long and 0.5 mm wide; its resistance is close to 100 ohms and the deposit is obtained either by painting or by cathode sputtering on a chromium underlayer. The terminals (3) consist of thinsilver layers which end on the lower surface of the calorimetric element where the output wires are soldered (4).

These wires are connected to a shielded connector through two sealed glass beads (5). The fluxmeter housing (6) made of brass is designed for mounting on the shock tube; it is provided with a base (7) to which the shielded connector is attached. This brass housing acts as a base for the calorimetric element which is surrounded with a plexigass sheath (8) to electrically insulate the silver terminals. The calorimetric element with its sheath is attached by means of a cellulose adhesive inside the pickup housing so that the sensing part can be replaced once damaged. The glass beads provide for a sealed vacuum for pressures up to 5 bars; to avoid a pressure differential between the two sides of the calorimetric element, a slot is made in the plexiglass sheath.

Good continuity between the wall of the shock tube and the pickup surface is necessary if the boundary layer is not to be disturbed. Still, the sharp angle must be removed at the edge of the sensing surface to prevent breakage of the thin silver layers which constitute the terminals; for this reason, the edges are rounded slightly with a radius of 0.3 mm. Tests have shown that these rounded edges were uncapable of causing a transition in the laminar boundary layers.

Wall fluxmeters are also used to measure the velocity of the shock according to the time elapsed between the thermal signals coming from two pickups separated by a known distance. Consequently, in order to best define the passing of the wave, it is important for the thermometer elements to be placed perpendicularly with respect to the tube's axis.

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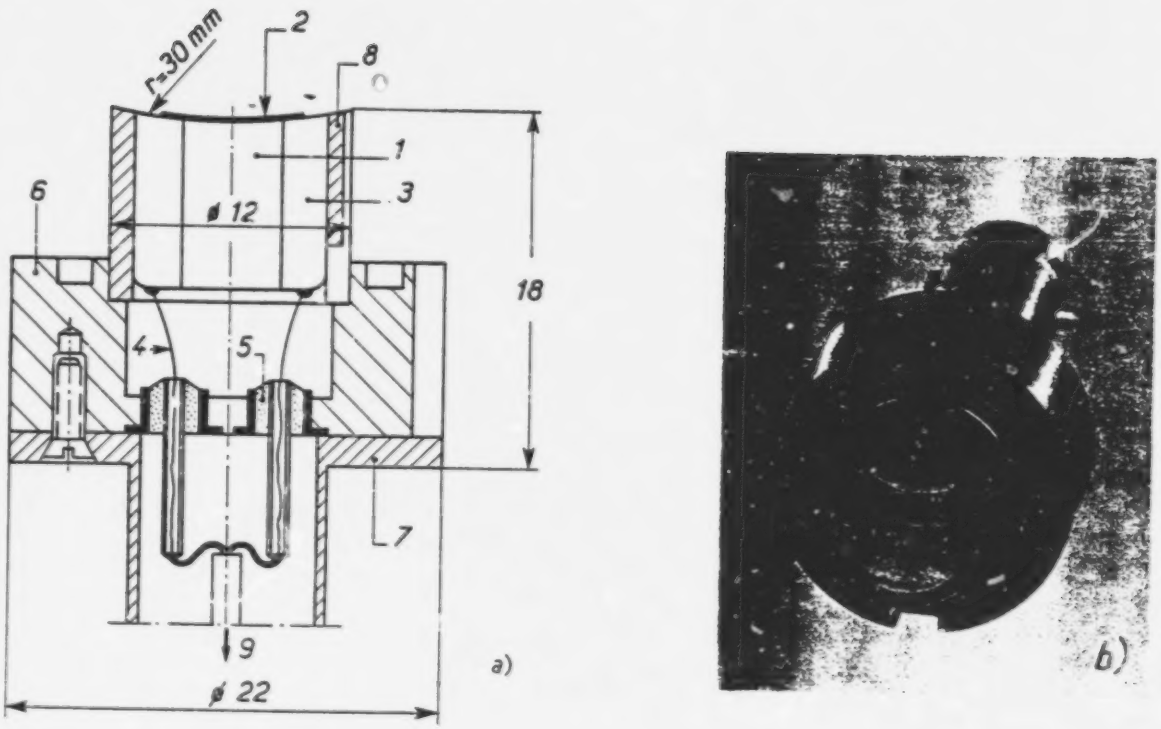

Figure 2 - Fluxmeter type "20 L 101"

1 - Silica calorimetric element
2 - Thermometer element
3 - Silver painted terminals
4 - Wire
5 - Glass beads
6 - Brass housing
7 - Base
8 - Plexiglass sheath
9 - To RADIAL I connector 4

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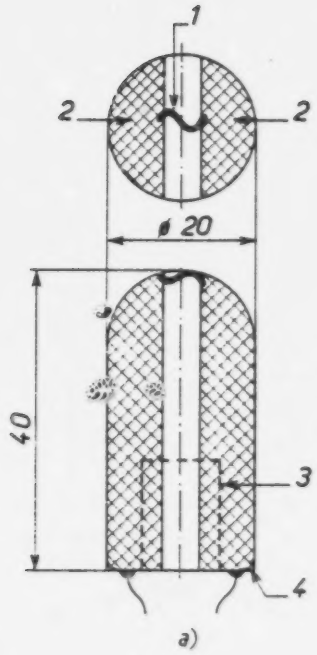

Figure 3 - Model for measuring impact flux

1 - Platinum thermometer element

2 - Silver painted terminals

3 - Hole for fastening, 10 mm diameter

4 - Rounded edges

V.2 MODELS FOR MEASURING THE IMPACT FLUX

The models (figure 3) are made entirely of silica and thus constitute the calorimetric element. They have a hemispherical portion with a radius of 10 mm, prolonged by a cylinder of the same radius. In the area of the impact point, a thin platinum layer (2) applied by painting measures the surface temperature; the platinum line does not deviate by more than 10 to 15 degrees from the impact point. The terminals are made from silver deposits (2) which extend to the base of the model.

So far, the cathode sputtering technique has not provided sufficiently adhesive deposits to withstand the erosion which takes place at the impact point.

V.3 CROSSWISE SHOCK WAVE DETECTORS

These pickups (figure 4) do not take actual flux measurements. Their only purpose is to deliver deliver a signal at the beginning of the flow in the axis of the shock tube. They have brass housings identical to those of the wall fluxmeters;

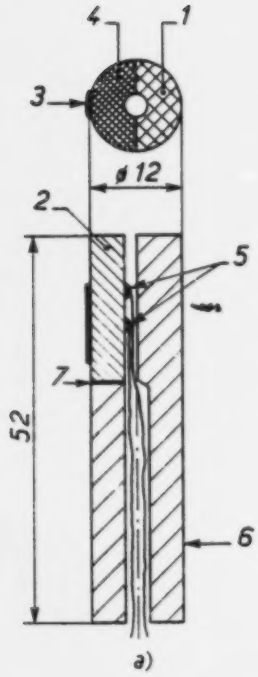

Figure 4 - Crosswise shock wave detector

1 - Steel base

2 - Calorimetric element

3 - Platinum thermometer element

4 - Silver painted terminal

5 - Junction wire solder joints

6 - Installation by gluing to housing

7 - Insulating paper

on the housing is mounted a steel cylindrical piece (1) having a diameter of 12 mm and sufficiently long to extend beyond the axis of the tube. At the end this piece is reduced to a half cylinder to which is bonded a silica calorimetric element having the same shape (2). A line of platinum paint (3) is deposited on the impact generating line. The junctions are ensured by thin deposits of silver (4).

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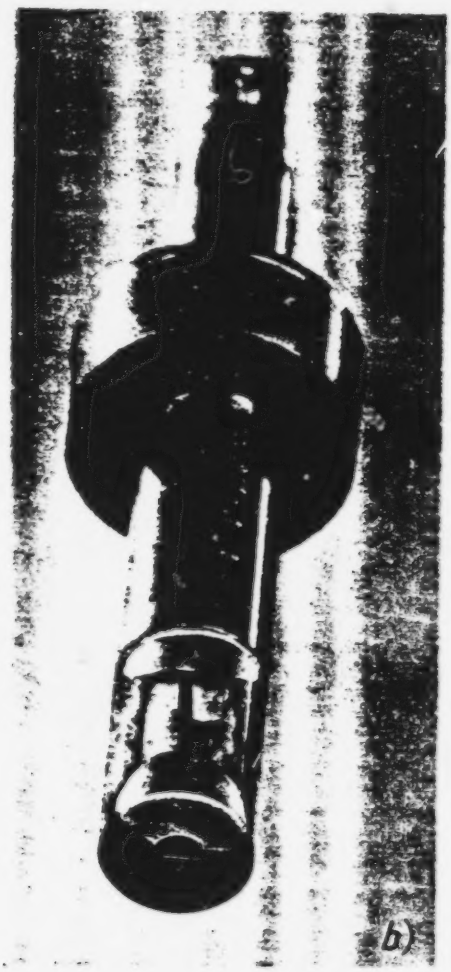

Figure 4b

VI. CALIBRATIONS

Equation 5 is written while taking into account the variable resistance R of the thermometer element:

$$\varphi = \frac{1}{\sqrt{\tau}} \frac{\sqrt{\lambda \rho c}}{R_0 \alpha} \int_0^{\tau} \frac{dR}{d\tau} \frac{d\tau}{\sqrt{t - \tau}} \qquad (13)$$

The purpose of calibrating a fluxmeter is to determine the coefficient $\frac{\sqrt{\lambda \rho c}}{R_0 \alpha}$, whose numerator expresses the thermal properties of the calorimetric element and whose denominator those of the surface thermometer.

Before describing the different methods used to

measure the coefficient of a fluxmeter, it is important to define the values of the electrical current that can be used. If the thin layer thermometer were intended to take a simple temperature measurement, the current would have to be limited to a sufficiently low value so that this temperature is not noticeably disturbed by the Joule effect. When measuring thermal flux this limitation does not exist in theory because, with the heat equation being linear, equation 13 remains valid regardless of the initial temperature distribution. Nevertheless, we have found that the use of too high a current - in excess of about 10 milliamperes - was accompanied by interfering phenomena whose causes have not yet been explained. For example, if the resistance is first measured with a current of 500 microA, and then this current is suddenly increased to 50 mA, we witness an instantaneous variation in this resistance which is independent of the thermal effect which occurs through progressive heating. This effect - already reported by other authors [10] - is perhaps

connected with the very high current densities in the thin layers on the order of a mega-ampere per square centimeter; it is not impossible that the electrical properties, such as the resistivity and the temperature coefficient, undergo more or less extensive modifications.

In a critical examination of the calibration methods, we must take into consideration the drawbacks brought on by the use of high currents. These methods shall be described and their results compared.

VI.1 SEPARATE MEASUREMENTS OF THERMAL PROPERTIES

The most elementary process consists in determining the coefficient $\sqrt{\lambda\rho c}$ of silica and the thermal sensitivity of the resistance, expressed by $R_0\alpha$, separately. The validity of this method is disputed by a certain number of authors who give the following reasons: during a shock tube test the thermal signal only penetrates the calorimetric element to a depth of about 0.1 mm. But the properties of the silica in this surface layer are different from those of the core material, since the bonding of the thermometer film implies some diffusion of the metal into the silica and thus interferes with the thermal conductivity [11]. But there are authors who share the opposite opinion [10] [3]; this second opinion seems justified: indeed, it is difficult to imagine diffusion of the metal to a depth much greater than the thickness of the thin layer. In addition, if such diffusion did exist the thermal conductivity would only be able to vary with the distance from the surface and would not provide a constant heating under the flux proportional to $\sqrt{t}$, as has always been the case in the experiments. We must therefore assume that the coefficient $\sqrt{\lambda\rho c}$ involved in the measurement is identical to that which can be determined for a solid sample of silica.

Unfortunately, thermal conductivity measurement attempts made upon ONERA request by a specialized laboratory did not arrive at correct results. The comparison of the values provided with those yielded by other calibration methods reveal sizable random errors. What's more, we have seen that the values given in the physical constant tables were to be taken with a grain of salt. This is why it is necessary to consider other processes for measuring the coefficient.

However, determination of $R_{\theta\alpha}$ does not involve any difficulty. We use a thermostatic bath which, for instance, can be adjusted between ambient temperature and 40 degrees C. Within this range the law that links the resistance to the temperature is perfectly linear and the influence of a quadratic term is less than the measurement error. Thanks to this method the product $R_{\theta\alpha}$ can be determined to within less than plus or minus 1%.

VI.2 SIMPLE ELECTRICAL METHOD

The calibration method universally adopted by laboratories consists in subjecting the pickup to the heat flux generated by a strong current in the thermometer element [11] [3]. This process was implemented in particular by the Franco-German Research Institute in Saint-Louis [12] and the ONERA for former testing in brief flows [13].

The amperage must be sufficient to cause a noteworthy temperature rise for a duration approximating that of a shock tube test. The variation in resistance, which constitutes the variable branch of a Wheatstone bridge, is recorded with a fast oscillograph. If the current I remains relatively constant, then equation 4 gives us the following expression for the fluxmeter coefficient:

(14) $$\frac{\sqrt{\lambda\rho c}}{R_0\alpha} = \frac{2RI^2\sqrt{t}}{\sqrt{\pi}\cdot S\Delta R}$$

where S is the surface of the thermometer film and $\frac{\Delta R}{\sqrt{t}}$ the slope of experimental straight line obtained by plotting the resistance variations along the y-axis and the square roots of the corresponding times along the x-axis.

In its work configuration the electrical circuit provides a heating current of 50 mA through the resistor in the form of a 500 micro-s strobe. At rest, the current traveling through the thin layer is limited to 500 micro-A.

Under the effect of the high current we have seen that an instantaneous imbalance in the bridge generally appears at the beginning of the test. In addition, a pulse is recorded at that moment due to a slight initial imbalance caused by the imaginary portion of the impedances. This is not a drawback for the measurement provided that we ignore the beginning of the recording when the transient phenomena connected with the fluxmeter's response time also occur.

Some precautions are necessary to prevent a considerable portion of the heat flux from dissipating into the surrounding air. The pickup must be shielded from air currents and the sensing surface pointed downward in order to limit natural convection. Under these conditions the effect of the air on the surface heating is very small. This heating is proportional to the flux density φ_0, assumed to be constant; the relative error is therefore approximately equal to the ratio $\frac{\varphi_a}{\varphi_0}$, where φ_a represents the portion of the flux dissipated into the air, which is very small compared to φ_0 ; this ratio is equal to $\frac{(\sqrt{\lambda\rho c})_{air}}{(\sqrt{\lambda\rho c})_{silica}}$.

For the amibient temperature we have: $(\sqrt{\lambda\rho c})_{air} =$ 0.54 x 10-3

J/cm2 degrees-C-s1/2; when compared with the corresponding value for silica - that is, about 0.154 - the error in the surface temperature rise is 0.35%. If a term connected with the natural convection is added to φ_0, we arrive at an additional error of the same magnitude, if we take the coefficient accepted for a flat surface directed downward and for temperature differences close to those obtained during electrical calibration. The total error is therefore still between 0.5% and 1% and vacuum calibration does not appear to be necessary.

Also, since the flux is only generated in the thin layer, the heat equation loses its

single-dimensional characteristic as soon as the penetration depth of the thermal signal becomes substantial compared to the width of the thermometer element, assumed to be rectangular. A correction of the measured values of the temperature rise is therefore necessary; the calculation of this correction is dealt with in Appendix 4: it shows that the deviation of the change in Ts with a parabolic law is not negligible after 500 microseconds for thin layers 0.5 mm wide. The experiment showed that, once the theoretical correction is applied, the variation of R with $\sqrt{t}$ becomes almost linear.

In spite of all these precautions, numerous electrical calibration tests conducted at the ONERA yielded unacceptable values for the coefficients. If we calculate $\sqrt{\lambda\rho c}$ using equation 14 and using an $R_{0\alpha}$ determined earlier with a thermostatic bath, most of the values show a considerable dispersion and also appear much lower (at least 30% on the average) than those taken from recent publications. In these publications, apparently reporting very consciencious

measurements, the data are more tightly grouped than if we consider all of the tables, some of which are quite old.

The imprecision connected with measurement of the sensing element surface is the cause for part of the dispersion observed. This does not explain why the calculated values of $\sqrt{\lambda\rho c}$ are always lower than the normal ones.

A deeper examination of the method can demonstrate theoretically that the heterogeneities in the thin layer, such as the thickness inconsistencies, are the cause for this anomaly. The conclusions of the calculation presented in Appendix 5 contradict those of J.R. Busing [14], the only author who seems to have foreseen this problem. According to this publication the thickness of a thin layer applied by painting varies in a section of the layer according to a parabolic law. We let z(x) be this thickness, a function of an x coordinate governed by the width of the thermometer element. The exact theory results in the following ratio of the coefficient calculated through electrical calibration to the true coefficient:

(15) $$\frac{(\sqrt{\lambda\rho c})_{calculé}}{\sqrt{\lambda\rho c}} = \frac{\left(\int z dx\right)^2}{l\int z^2 dx}$$

where the integrals are extended to the width l of the thin layer. However, for a z variation of parabolic form, equation 15 is equal to 5/6, whereas in reference [14] an identical value is assumed for both the measured coefficient and the true coefficient.

The equation can only be followed through from simple diagrams expressing the geometric irregularities of the thin layers. However, the real heterogeneities are of a random nature. The theory, therefore, provides only one qualitative

justification for the results obtained by showing that the calculated coefficient must always be smaller than its true value. As a consequence, the electrical method as just described, does not appear applicable if highly uniform thin layers are not available; moreover, the possible variations in the properties of the resistance when subjected to the strong currents are not taken into account; such variations would probably tend to increase the error made.

This process only seems worthy of being adopted as a comparative means of evaluating the stability with time of a thin layer, with any variation in R_{ox} having to result in a proportional variation in the pickup response in which a constant electrical power is dissipated.

VI.3 DOUBLE ELECTRICAL METHOD

Among the drawbacks of the method just described, the difficulty in accurately measuring the surface of the thin layer has especially captivated the attention of certain experimenters; these experimenters proposed a double electrical method which consists in making two measurements of the heating caused by Joule effect, one in the same way as already described, the other after having dipped the surface of the fluxmeter in a still liquid of known thermal properties; the power RI2 dissipated must be the same in both tests.

Let us take equation 14 in which the second term is multiplied by a theoretically known factor A and which takes into account the influence of the heterogeneities of the thin layer. We let $\Delta R_1/\sqrt{i}$ be the slope of the straight line obtained from the test in air, and $\Delta R_2/\sqrt{i}$ that of the straight line corresponding to the test in the liquid whose thermal properties are expressed by $(\sqrt{\lambda\rho c})_{liquide}$. In the second test a fraction m

of the heat flux is dissipated in the liquid; we can therefore write three equations:

$$(16)\qquad \begin{cases} \dfrac{(\sqrt{\lambda\rho c})_{\text{liquide}}}{R_0\alpha} = Am\,\dfrac{2\,RI^2}{\sqrt{\pi}\,S\,(\Delta R_2/\sqrt{t})} \\ \dfrac{\sqrt{\lambda\rho c}}{R_0\alpha} = A\,(1-m)\cdot\dfrac{2\,RI^2}{\sqrt{\pi}\,S\,(\Delta R_2/\sqrt{t})} \\ \dfrac{\sqrt{\lambda\rho c}}{R_0\alpha} = A\cdot\dfrac{2\,RI^2}{\sqrt{\pi}\,S\,(\Delta R_1/\sqrt{t})}. \end{cases}$$

From equaticn system 16 we obtain:

$$(17)\qquad \sqrt{\lambda\rho c} = (\sqrt{\lambda\rho c})_{\text{liquide}}\cdot\frac{1}{\dfrac{\Delta R_1/\sqrt{t}}{\Delta R_2/\sqrt{t}} - 1}.$$

Knowing $(\sqrt{\lambda\rho c})_{\text{liquide}}$, we can deduce the corresponding value for the silica. The surface of the thin layer as well as the effect of the heterogeneities in its shape are eliminated from equation 17. But this method does not provide any information concerning $R_0\alpha$, which must be determined with a thermostatic bath.

Assuming that the random errors made in $\Delta R_1/\sqrt{t}$ and $\Delta R_2/\sqrt{t}$ are identical, we can show through calculation that the optimum value of the ratio of these two slopes (making the error in $\sqrt{\lambda\rho c}$ as small as possible) is equal to $(1+\sqrt{2})$. Theoretically, the best liquid would be one in which: $\dfrac{(\sqrt{\lambda\rho c})_{\text{liquide}}}{\sqrt{\lambda\rho c}} = \sqrt{2}$.

But other considerations must also come into play in the choice of a liquid. The thermal properties of water are particularly well known. This is why several authors have suggested its used [10] [15]. Nevertheless, tests have shown that the presence of water, even distilled water, interfered with the resistance measurement: perhaps the thermometer element should be covered with a non-conducting thin layer. This operation would be necessary anyway for flux measurement in a flow having a very high Mach number which therefore implies a high rate of ionization. To accomplish this certain laboratories have used magnesium fluoride [10] or silicon monoxide [9]. Nevertheless, the problem of the electrical insulation of the

thin layer has not yet been raised at the ONERA. For this reason another liquid, glycerol, was used. Its thermal properties are known with much precision. According to the data in the tables, it seems possible to assume a value of 0.0925 J/cm2 degrees-C-s1/2 for $(\sqrt{\lambda\rho c})_{\text{liquide}}$ with a precision of about plus or minus 4%. Glycerol is a perfect electrical insulator - a considerable advantage - even though the ratio $\frac{(\sqrt{\lambda\rho c})_{\text{liquide}}}{(\sqrt{\lambda\rho c})_{\text{silice}}}$, which is close to 0.6, is quite far from the optimum value of $\sqrt{2}$.

The random error in the silica coefficient, calculated from equation 17, is high. It is assumed that the slopes $\Delta R\sqrt{\tau}$ are known to within plus or minus 2%, taking into consideration both the measurement errors of the resistance and the time, and those resulting from the plotting of straight lines. The reproducibility of the electrical power RI2 in both tests can be ensured to within plus or minus 1%. In the case of calibration this gives a random error of 16% to which must be added that coming from the uncertainty in our knowledge of the thermal properties of glycerol.

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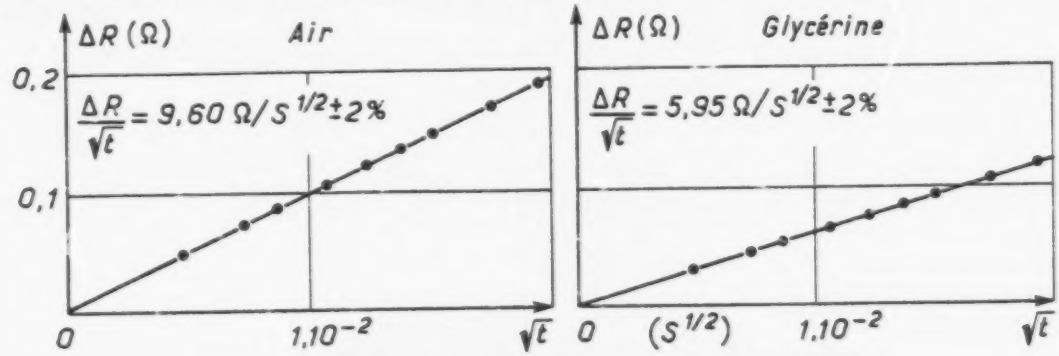
For a given silica sampling, this error calculation is liable to produce considerable dispersion in the measurements. Such dispersion was correctly checked quantitatively by experimentation; also, we must only consider the average of very numerous determinations of $\sqrt{\lambda \rho c}$.

Figure 5 represents a group of air-glycerol calibrations; the oscillograms take on the shape of modified parabolas at the end of the test by the lateral diffusion of the heat. The curves giving ΔR as a function of $\sqrt{t}$, are very properly linear after correction for this interfering effect.



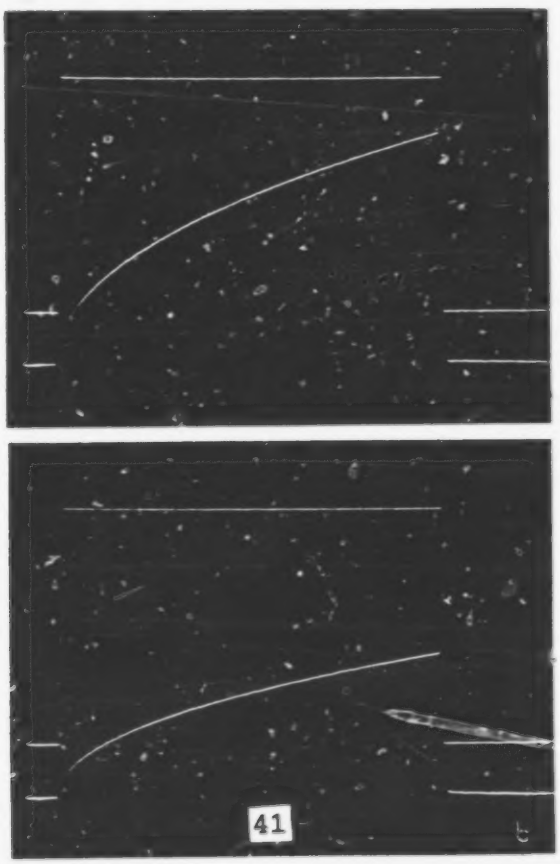


Figure 5 - Electrical calibration of a fluxmeter

a - Test in air

b - Test in glycerol

$$R = 103{,}1\ \Omega. - R_{\theta\alpha} = \frac{dR}{dt} = 0{,}114\ \Omega/^{\circ}C \pm 1\ \%.$$
$$I = 56{,}5\ mA. - RI^2 = 0{,}329\ watt \pm 1\ \%.$$
$$\sqrt{\pi\lambda\rho c} = 0{,}268.$$

Use of test in air alone: surface of thermometer element corrected. S' = 4.6 mm2

$$\sqrt{\pi\lambda\rho c} = 2\frac{RI^2}{S'} \cdot \frac{R_{\theta\alpha}}{\Delta R/\sqrt{t}} = 0{,}170.$$

We shall discuss the results obtained from 48 double tests performed on 24 pickups all taken from the same bar of silica (two calibration groups per pickup). If we let the random portion of the maximum error be equal to

13

16% and the distribution normal, the average value taken from the 48 tests must be fraught with an uncertainty in the vicinity of 2%, to which is added the uncertainty in our knowledge of the thermal properties of the liquid. All in all, this gives plus or minus 6%. The value obtained for $\sqrt{\lambda\rho c}$ is equal to 0 143 J/cm^2-degreesC-s$^{1/2}$ to with an average of about 5 to 6% for the absoluate values of the relative deviations.

In reference [8] a value of $\sqrt{\lambda\rho c}$ equal to 0.154 J/cm^2-degrees-C-s$^{1/2}$ is proposed within plus or minus 1%; the deviation from the results of the double electrical calibrations becomes 7%. This deviation is not entirely incompatible with the calculated errors. Nevertheless, we may wonder whether the phenomena accompanying the strong electrical current in the thin layers run the risk of vitiating the calibrations to a certain extent. For this reason another method, which does not seem to have been used thus far, was designed and applied with varying degrees of success.

VI.4 THERMAL RADIATION METHOD

This method consists in subjecting the pickup to a known thermal radiation. The flux is delivered by a set of six tungsten filament tubes mounted in an aluminum reflector. The assembly consumes a power of 11 kW; the thermal flux is made uniform using a satin-finished silica plate measuring 30 x 15 cm. The pickup is placed on a plate cooled by circulating water and a moving shutter also cooled allows it to be uncovered for the desired time. The value of the thermal flux is adjusted according to the distance separating the pickup from the reflector; for this purpose the source can be placed vertically and its position carefully identified. The maximum value of the flux density that can be obtained using this type of generator is 25 W/cm^2.

The flux is determined using a thermal inertia calibration device which is first placed in the same place as the fluxmeter to be tested. Studies with the calibrator have allowed us to define its position to within plus or minus 3%. The entire device is described in detail in reference [16].

In order to ensure an identical value for the fluxes determined by the calibrator pickup and by the pickup subjected to the tests, the absorption factors of the exposed surfaces must be the same. The process chosen consists in covering these two surfaces with a layer of carbon black which must be thick enough for the silica pickup so that no radiation enters the transparent material. The experiment showed that this condition is correctly fulfilled when the deposit reaches a value of 0.25 mg/cm^2, which corresponds approximately to one micron of pure carbon.

This thickness forces the fluxmeter to have a response time which is not regligible; nevertheless, since the test is analyzed

over a period of 0.25 second, the response time is not a problem. At the end of the experiment the thermal signal penetrated the silica to a depth of about 2 millimeters, slightly deeper than in a shock tube test. The measurement method is still valid, however, since the thermal properties of the material remain constant in the solid material.

The measurement system comprises a Wheatstone bridge and a photographic recording oscillograph. At the same time the signal comes from the pickup, that from a reference chronometer at a frequency of 20 Hz is recorded to determine the value of the time base. The current supplying the thermometer element is close to a milliampere; such a small current is not in danger of interfering with the thin layer properties.

Using equation 4 we can arrive at the pickup coefficient from the resistance variation when the density of the flux absorbed φ_0 is constant:

$$\frac{\sqrt{\lambda \rho c}}{R_{\varphi x}} = \frac{2 \varphi_0 \sqrt{t}}{\sqrt{\pi} \cdot \Delta R} \qquad (18)$$

The uncertainty in φ_0 is plus or minus 3% and that affecting $R_{\varphi x}$ is plus or minus 1%. The error made on the slope of the straight line plotted as a function of $\sqrt{t}$ can attain a maximum of plus or minus 6%. The latter error has several causes: firstly, the alignment error of the points resulting from uncertainties in the elongation readings; secondly, the error made in the marking at the initial heating instant; lastly, the time during which the pickup is uncovered by the shutter: the shutter is controlled by an electromagnet and its speed is close to 70 cm/s. Setup of the thermal flux is not instantaneous and a calculation allows us to make an approximate correction to account for this.

Nevertheless, this calculation still leaves room for a non-negligible uncertainty.

In order to increase the accuracy of the calibration, other corrections must be made concerning the non-linearity effects of the bridge, the curvature of the pickup surface, the variations in the thermal properties of the silica with temperature, and a slight decrease in the flux caused by the surface heating and natural convection. Lastly, in the case of fluxmeters having a spherical surface, we must take into account a drop in the heat flux with respect to that which affects a flat surface, by applying the cosine law.

Once all these corrections are made, the points which translate the values of ΔR as a fujnction of $\sqrt{t}$ align correctly and the maximum random error on $\sqrt{\lambda\rho c}$ is close to plus or minus 10%. Figure 6 shows an oscillogram and the curve of the resistance variations deduced from it; to highlight the importance of the corrections the points obtained directly with the recording are also shown.

A series of 72 calibrations were conducted on the 24 pickups mentioned in the preceding chapter (3 tests per pickup).

In consideration of the fact that several values of thermal flux were used, the accuracy of the coefficient $\sqrt{\lambda\rho c}$ was evaluated at plus or minus 2% on the average. Each of the values was calculated from equation 18 and using the $R_{o\alpha}$ measured in the thermostatic bath before and after each calibration to ensure that this value did not vary noticeably.

The average value obtained was 0.155 J/cm^2-degreesC-$s^{1/2}$, that is 8% greater than that provided by the double air-glycerol electrical calibrations. The average of the absolute values of the relative deviations is about 4%. The coefficient determined by calibration under radiation for this sample of silica is very

close to that proposed in reference [8]. However, the author claims that silica bars of different origin yield very nearly the same results; but measurements were made at the ONERA on numerous pickups taken from 3 other samples of the material; the average values obtained for $\sqrt{\lambda\rho c}$ were greater than 0.155 J/cm^2-degreesC-s$^{1/2}$ by 3.3%, 1.1%, and 4.1% respectively.

Note that no characteristic difference could be brought out for pickups whose surfaces were

of different shapes (flat, cylindrical, or spherical), nor for different types of thin layers (painted deposits, platinum cathode sputtering deposits, or nicket cathod sputtering deposits).

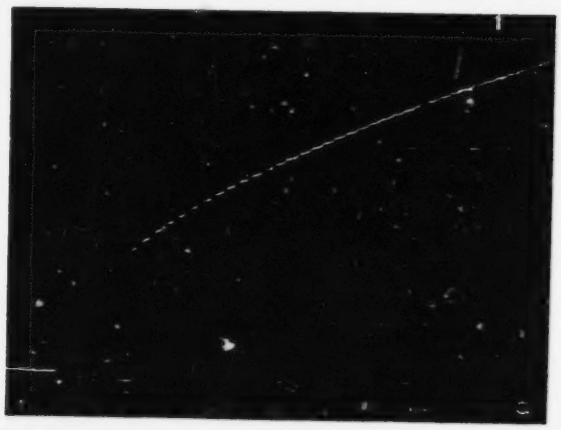


Figure 6 - Calibration of a fluxmeter under radiation flux

X corrected values

• uncorrected values

$R_0 = 151\ \Omega.\ \frac{\Delta R}{\sqrt{t}} = 13{,}4\ \Omega/s^{1/2} \pm 6\ \%.\ \varphi_0 = 6{,}0\ W/cm^2 \pm 3\ \%.$

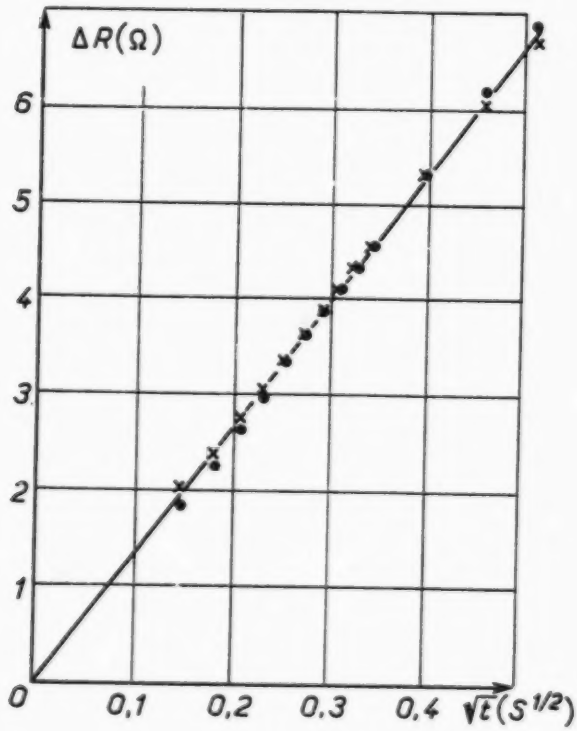
$R_{0x} = \frac{dR}{dt} = 0{,}338\Omega/°C.\quad \sqrt{\pi\lambda\rho c} = \frac{2\varphi_0 R_{0x}}{\Delta R/\sqrt{t}} =$

$0{,}303\ J/cm^2\text{-}°C\text{-}s^{1/2}\ (\pm 10\ \%).$

The deviations observed between the thermal properties of several silica bars are partly attributable to the experimentation errors, but they also seem to confirm very slight variations from one sample to another.

The values obtained by calibration under radiation are therefore close to those proposed in the most recent publications. In addition, the maximum random error resulting from the calculation is lower than for double electrical calibration. Lastly, the use of very small currents in the thin layers does not run the risk of bringing about possible modifications in their properties.

The thermal radiation method thus seems to be the most reliable for calibrating surface temperature fluxmeters and the coefficients that it has provided have been used for analyzing shock tube tests.



VII. CONCLUSION

The surface temperature fluxmeter belongs to a category of devices commonly used by aerodynamics specialists. However, the problem of calibrating these pickups had not yet found entirely satisfactory solutions. A study conducted at the ONERA led to the designing of fluxmeters for the pilot shock tube in Chalais-Meudon and the perfecting of a new calibration process which appears more accurate than the usual methods based on electrical heating. A simple calculation leads to the conclusion that the latter methods are inaccurate when the structure of the thin layer is not perfectly homogeneous. In order to make such layers the process of cathode sputtering is preferably to the well-known painting method, at least in regard to stability; however, the thickness still isn't perfectly uniform regardless of the means of depositing.

Further testing will be conducted to improve the thin layer fabrication methods and to increase the accuracy of the calibrations.

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APPENDIX 1

INFLUENCE OF THE SURFACE CURVATURE

We attempt to find the solution to the heat equation in the two cases corresponding to the pickups made, whose surfaces are either convex spherical or concave spherical.

a) Case of the spherical surface

The diffusion equation is written:

(1) $$a\left(\frac{\partial^2 T}{\partial x^2} - \frac{2}{r-x}\frac{\partial T}{\partial x}\right) = \frac{\partial T}{\partial t}$$

where T is the temperature rise at a distanc x from the surface whose curvature radius is equal to r. The thermal diffusivity is represented by $a = \frac{\lambda}{\lambda c}$.

The symbol ζ represents the Laplace transform of the function ((r - x)T. We then have:

(2) $$a\frac{\partial^2 \zeta}{\partial x^2} = p\zeta.$$

At the center of the sphere (x = r) ζ is nil. At the surface (x = 0) we apply the heat flux whose density φ_0 is assumed to be independent of time:

(3) $$\varphi_0 = -\lambda\frac{\partial T}{\partial x}.$$

After transformation the condition expressed by equation 3 becomes:

(4) $$\frac{\varphi_0}{p} = -\lambda\frac{\partial}{\partial x}\left(\frac{\zeta}{r-x}\right).$$

In consideration of the foregoing remarks the solution to equation 2 is written as:

(5)

$$\zeta = \frac{\varphi_0 r}{p\lambda} \frac{e^{z\sqrt{\frac{p}{a}}r} e^{-\sqrt{\frac{p}{a}}x} - e^{\sqrt{\frac{p}{a}}x}}{\frac{1}{r}\left(1 - e^{z\sqrt{\frac{p}{a}}r}\right) + \sqrt{\frac{p}{a}}\left(1 + e^{z\sqrt{\frac{p}{a}}r}\right)}.$$

If Ts is the surface temperature rise, ζ (x = 0) is the transform of rTs. We can therefore express the transform θ_s of Ts by:

(6)

$$\theta_s = \frac{\varphi_0}{\sqrt{\lambda\rho c}\, p^{3/2}} \cdot \frac{1 - e^{-z\sqrt{\frac{p}{a}}r}}{\left(1 - \frac{1}{r}\sqrt{\frac{p}{a}}\right) + e^{-z\sqrt{\frac{p}{a}}r}\left(1 + \frac{1}{r}\sqrt{\frac{p}{a}}\right)}$$

$$= \frac{\varphi_0}{\sqrt{\lambda\rho c}\, p^{3/2}} \left[\sum_{n=0}^{\infty} \left(\frac{1}{r}\sqrt{\frac{a}{p}}\right)^n + 2\sum_{n=1}^{\infty} (-1)^n e^{-2n\sqrt{\frac{p}{a}}r} \left(1 + 2n\frac{1}{r}\sqrt{\frac{a}{p}} + [2n^2+1]\frac{1}{r^2}\frac{a}{p} + \cdots\right)\right].$$

The inversion of θ_s developed in series according to the above formula provides the value of the surface temperature rise:

(7)

$$(7)\quad T_s = \frac{2\varphi_0\sqrt{t}}{\sqrt{\pi\lambda\rho c}}\left[1 + \frac{\sqrt{\pi}}{2}\cdot\frac{\sqrt{at}}{r} + \frac{2}{3}\cdot\frac{at}{r^2} + \cdots + 2\sum_{n=1}^{\infty}(-1)^n\left(e^{-n^2\frac{r^2}{at}}\left[1 + \frac{4n^2}{3}(n^2-1) + \frac{2}{3}(2n^2+1)\frac{at}{r^2} + \cdots\right] - \sqrt{\pi}\cdot n\,\mathrm{erfc}\left(n\frac{r}{\sqrt{at}}\right)\left[2n^2\frac{\sqrt{at}}{r} + \frac{r}{\sqrt{at}} + \frac{4n^2}{3}(n^2-1)\frac{r}{\sqrt{at}} + \cdots\right]\right)\right].$$

To simplify this expression we can see that the term acting as a factor in the second member represents the surface heating of a flat surface pickup. The deviations from this value depend on the dimensionless quantity $\sqrt{at}/r$ which, to within a digital factor, represents the ratio of the penetration depth of the

thermal signal to the curvature radius. However, for the measurement, it is indispensable for this ratio to be small enough with respect to unity. For $\frac{nr}{\sqrt{at}}$ sufficiently large, we can develop the complementary error function in the following form:

(8)
$$\operatorname{erfc}\left(n\frac{r}{\sqrt{at}}\right) = \frac{e^{-n^2\frac{r^2}{at}}}{\sqrt{\pi}}\left(\frac{1}{n}\frac{\sqrt{at}}{r} - \frac{1}{2n^3}\frac{[at]^{3/2}}{r^3} + \frac{3}{4n^5}\cdot\frac{[at]^{5/2}}{r^5} + \cdots\right).$$

By factoring expression 8 into 7 and ignoring the powers greater than $\frac{at}{r^2}$, there remains:

(9)
$$T_s = \frac{2\varphi_0\sqrt{t}}{\sqrt{\pi\lambda\rho c}}\left[1 + \frac{\sqrt{\pi}}{2}\frac{\sqrt{at}}{r} - \frac{at}{r^2}\left(\frac{2}{3} + 2\sum_{n=1}^{\infty}\frac{(-1)^n}{2n^2}e^{-n^2\frac{r^2}{at}}\right)\right].$$

From equation 9 we can determine the influence of the curvature during calibration of the silica pickups under a radiation flux. We have seen that the analysis of the test was performed up to a time of 0.25 second after the beginning of the signal. The curvature radius of the hemispherical models is equal to 1 cm and the diffusivity of silica is close to 10^{-2} cm^2/s. Hence:

$$\frac{r}{\sqrt{at}} = 20.$$

The first term under the Σ sign in 9 is perfectly negligible compared to 2/3; this is even more valid for shorter times. The correction factor to be applied to the measured temperature in order to convert to the case of a flat surface is therefore reduced to:

$$\frac{1}{1 + \frac{\sqrt{\pi}}{2}\frac{\sqrt{at}}{r} + \frac{2}{3}\frac{at}{r^2}}$$

In the numerical example considered this factor is equal to 0.954. But in a shock tube test with a duration of less than one millisecond the correction appears negligible.

b) Case of a cylindrical surface

We start with the following equation:

(10) $$a\left(\frac{\partial^2 T}{\partial x^2}+\frac{1}{r+x}\frac{\partial T}{\partial x}\right)=\frac{\partial T}{\partial t}.$$

If $\theta(p)$ is the Laplace transform of T:

(11) $$a\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{1}{r+x}\frac{\partial \theta}{\partial x}\right)=p\theta.$$

The general solution tc 11 is of the following form:

(12) $$\theta = A\cdot K_0\left(\sqrt{\frac{p}{a}}[r+x]\right)+B\cdot I_0\left(\sqrt{\frac{p}{a}}[r+x]\right)$$

where I_o and K_0 are the Bessel imaginary functions of order zero, and of the first and second species, respectively.

The condition of a semi-unlimited medium requires that θ be nil when x is infinite. At the surface equation 3 remains valid. If we take into account the following equality:

(13) $$\frac{\partial}{\partial x}K_0\left[\sqrt{\frac{p}{a}}(r+x)\right]=-\sqrt{\frac{p}{a}}K_1\left[\sqrt{\frac{p}{a}}(r+x)\right]$$

where K_1 is a function of the second species, and due to I_o tending toward infinity for infinite x (which forces us to take B as nil) the solution to 12 for x = 0 is written as:

(14) $$\theta_s = \frac{\varphi_0}{\sqrt{\lambda\rho c}\, p^{3/2}} \cdot \frac{K_0\left(\sqrt{\frac{p}{a}}\, r\right)}{K_1\left(\sqrt{\frac{p}{a}}\, r\right)}.$$

The asymptotic developments of K_0 and K_1 for $\sqrt{\frac{p}{a}}\, r$ sufficiently large, that is, for $\sqrt{\frac{at}{r}}$ being small compared to unity, are the following:

(15)
$$\begin{cases} K_0\left(\sqrt{\frac{p}{a}}\, r\right) = \\ = \sqrt{\frac{\pi}{2\sqrt{\frac{p}{a}}\, r}} \cdot e^{-\sqrt{\frac{p}{a}}\, r} \left(1 - \frac{1}{8\sqrt{\frac{p}{a}}\, r} + \frac{9}{128\frac{p}{a} r^2} \cdots\right) \\ K_1\left(\sqrt{\frac{p}{a}}\, r\right) = \\ = \sqrt{\frac{\pi}{2\sqrt{\frac{p}{a}}\, r}} \cdot e^{-\sqrt{\frac{p}{a}}\, r} \left(1 + \frac{3}{8\sqrt{\frac{p}{a}}\, r} - \frac{15}{128\frac{p}{a} r^2} \cdots\right). \end{cases}$$

If we bring it into equation 14:

(16) $$\theta = \frac{\varphi_0}{\sqrt{\lambda\rho c}\, p^{3/2}} \left(1 - \frac{1}{2\sqrt{\frac{p}{a}}\, r} + \frac{3}{8\frac{p}{a} r^2} \cdots\right).$$

We obtain the expression of the surface temperature rise by inversion:

(17) $$T_s = \frac{2\varphi_0\sqrt{t}}{\sqrt{\pi\lambda\rho c}} \left[1 - \frac{\sqrt{\pi}}{4} \frac{\sqrt{at}}{r} + \frac{1}{4} \frac{at}{r^2} \cdots\right].$$

The predominant term of the corrective factor is negative, which shows than a concave surface heats up more slowly than a flat or convex surface. In the case of wall fluxmeters, the radius r is equal to 3 cm. After 0.25 second, the dimensionless quantity $\frac{r}{\sqrt{at}}$ is close to 60; the quadratic term is negligible

and the deviation with respect to the heating of a flat surface is 0.74%.

APPENDIX 2

RESPONSE TIME

The delay of a pickup is due to the fact that the average temperature T'_m of the thermometer element, and not the surface temperature T_s of the calorimetric element is measured. To simplify the calculation we shall assume that the thin layer has a uniform thickness and that it is deposited on a flat surface; λ', ρ' and c' shall represent its thermal conductivity, its density, and its specific heat, respectively. The thermal diffusivity of the metal is represented by:

$$a' = \frac{\lambda'}{\rho' c'}.$$

We can express the rises in temperature T' and T in the thin layer and in the insulating material respectively as a function of the distance x from the surface of the thermometer element (the surface of the calorimetric element corresponds to x = e):

(1)
$$\begin{cases} a' \dfrac{\partial^2 T'}{\partial x^2} = \dfrac{\partial T'}{\partial t} \\ a \dfrac{\partial^2 T}{\partial x^2} = \dfrac{\partial T}{\partial t}. \end{cases}$$

In the form of Laplace transforms, these equations become:

(2)
$$\begin{cases} a' \dfrac{\partial^2 \theta'}{\partial x^2} = p\theta' \\ a \dfrac{\partial^2 \theta}{\partial x^2} = p\theta. \end{cases}$$

The general solutions are written as follows:

(3)
$$\begin{cases} \theta' = A' e^{-\sqrt{\frac{p}{a'}}x} - B' e^{\sqrt{\frac{p}{a'}}x} \\ \theta = A e^{-\sqrt{\frac{p}{a}}x} + B e^{\sqrt{\frac{p}{a}}x}. \end{cases}$$

The coefficients A', B', A, and B are taken from the limit conditions:

- for x is infinite, $T = \theta = 0$ (semi-unlimited medium);
- for $x = \varepsilon : \lambda' \frac{\partial \theta'}{\partial x} = \lambda \frac{\partial \theta}{\partial x}$ and $\theta' = \theta$ (continuity of the heat flux and the temperature);
- for $x = 0$, $\Phi = -\lambda' \frac{\partial \theta'}{\partial x}$ where $\Phi(p)$ p) is the Laplace transform of the heat flux density φ .

We thus obtain for the expression of θ' , by letting:

$$\beta = \sqrt{\frac{\lambda \rho c}{\lambda' \rho' c'}};$$

(4)
$$\theta' = \frac{\beta}{\sqrt{\lambda \rho c}} \cdot \frac{\Phi}{\sqrt{p}} \left[e^{-\sqrt{\frac{p}{a'}}x} + \right.$$

$$\left. + \left(e^{-\sqrt{-\left(\frac{p}{a'}\right)}x} - e^{\sqrt{\frac{p}{a'}}x} \right) \frac{e^{-2\sqrt{\frac{p}{a'}}\varepsilon}}{\frac{1-\beta}{1+\beta} - e^{-2\sqrt{\frac{p}{a'}}\varepsilon}} \right].$$

The transform θ'_m of the average temperature rise T'_m of the thin layer is given by:

(5)
$$\theta'_m = \frac{1}{\varepsilon} \int_0^{\varepsilon} \theta' dx =$$

$$= \frac{\beta}{\sqrt{\lambda \rho c}} \frac{\Phi}{p} \frac{\sqrt{a'}}{\varepsilon} \left[1 - \frac{2\beta e^{-\sqrt{\frac{p}{a'}}\varepsilon}}{1 - \beta - (1-\beta) e^{-2\sqrt{\frac{p}{a'}}\varepsilon}} \right].$$

This formula can be written as follows by introducing the time characteristic:

(6)

$$t_c = \frac{e^2}{\beta^2 a'} = \frac{\rho'^2 c'^2 e^2}{\lambda \rho c}.$$

That is:

(7)

$$\theta'_m = \frac{1}{\sqrt{\lambda \rho c} \cdot \sqrt{t_c}} \cdot \frac{\Phi}{p} \left[1 - \frac{2\beta \cdot e^{-\beta \sqrt{pt_c}}}{1 + \beta - (1 - \beta) e^{-2\beta \sqrt{pt_c}}} \right].$$

Since the thermal conductivity λ of the insulating material is small compared to that of the metal λ' β is a quantity clearly under unity. We can therefore develop equation 7 in increasing powers of β, which yields:

(8)

$$\theta'_m = \frac{1}{\sqrt{\lambda \rho c} \sqrt{t_c}} \cdot \frac{\Phi}{p} \left[\frac{\sqrt{pt_c}}{1 + \sqrt{pt_c}} + \beta^2 \cdot \frac{pt_c \left(1 + \frac{\sqrt{pt_c}}{3} \right)}{2 (1 + \sqrt{pt_c})^3} + \cdots \right].$$

For a silica pickup having a platinum thermometer element the value of β is close to 0.11. We show that for the calculation of the response time, the term in β^2 can be considered negligible; this is equivalent to considering the thermal conductivity λ' of the metal as infinite, that is, the thin layer as isothermal in its thickness.

Expression 8 is reduced to:

(9)

$$\theta'_m = \sqrt{\frac{t_c}{\lambda \rho c}} \cdot \frac{\Phi}{\sqrt{pt_c} - pt_c}.$$

We shall consider the two cases of a constant flux $(\varphi = \varphi_0)$ and of a flux of the following type: $\varphi = \frac{m}{\sqrt{t}}$ which corresponds to an increment of surface temperature. The values of the measured temperature rise are given respectively by:

(10)

$$\begin{cases} \theta'_m = \varphi_0 \sqrt{\frac{t_c}{\lambda \rho c}} \cdot \frac{1}{p (\sqrt{pt_c} + pt_c)} \\ T'_m = \varphi_0 \sqrt{\frac{t_c}{\lambda \rho c}} \left[\frac{2}{\sqrt{\pi}} \sqrt{\frac{t}{t_c}} - 1 + e^{t/t_c} \cdot \operatorname{erfc} \sqrt{\frac{t}{t_c}} \right] \end{cases}$$

for a flux increment, and by:

(11)
$$\begin{cases} \theta'_m = m\sqrt{\pi}\sqrt{\frac{t_c}{\lambda\rho c}}\cdot\frac{1}{\sqrt{p}\,(\sqrt{pt_c}+pt_c)} \\ T'_m = \frac{m\sqrt{\pi}}{\sqrt{\lambda\rho c}}\left(1-e^{pt}\cdot\operatorname{erfc}\sqrt{\frac{t}{t_c}}\right) \end{cases}$$

for a surface temperature increment. In both cases the values of T_S that would be measured if the thermal inertia of the layer were nil are, respectively:

(12)
$$T_s = \frac{2\varphi_0\sqrt{t}}{\sqrt{\pi\lambda\rho c}}$$

and:

(13)
$$T_s = \frac{m\sqrt{\pi}}{\sqrt{\lambda\rho c}}.$$

We can calculate the relative error made by taking the values of T'_m instead of T'_s, that is:

(14)
$$\eta = \frac{T_s - T'_m}{T'_s}.$$

For a flux increment:

(15)
$$\eta = \frac{\sqrt{\pi}}{2}\cdot\frac{1-e^{pt}\cdot\operatorname{erfc}\sqrt{\frac{t}{t_c}}}{\sqrt{\frac{t}{t_c}}}$$

and for a temperature increment:

(16)
$$\eta = e^{pt}\cdot\operatorname{erfc}\sqrt{\frac{t}{t_c}}.$$

The characteristic time t_c of a silica fluxmeter with a platinum thin layer 0.1 micron thick is 3 x 10^{-8} second. If $\sqrt{\frac{t}{t_c}}$ reaches at least a value close to 20, then equation 15 reduces very nearly to:

(17)

$$\eta = \frac{\sqrt{\pi}}{2} \cdot \frac{1}{\sqrt{\frac{t}{t_c}}} = \frac{0,886}{\sqrt{\frac{t}{t_c}}}.$$

18

With $\sqrt{\frac{t}{t_c}}$ having a sufficiently large value, the asymptotic development of 16 yields:

(18)

$$\eta = \frac{1}{\sqrt{\pi}} \cdot \frac{1}{\sqrt{\frac{t}{t_c}}} = \frac{0,564}{\sqrt{\frac{t}{t_c}}}.$$

The calculation of the response time can be performed using equations 17 and 18 if the error allowed is less than 5%. The ratios $\frac{t}{t_c}$ are 280 and 7850 for η = 5% and η = 1% respectively, in the case of a constant flux. We obtain 134 and 3181 respectively, if we consider a surface temperature increment. In the numerical example: t_c = 3 x 10^{-8} second, this gives:

	Response time at 5%	Response time at 1%
Flux increment	8.4 microseconds	235 microsec
Temperature increment	4 microseconds	95 microsec

APPENDIX 3

INFLUENCE OF VARIATIONS IN THERMAL PROPERTIES WITH TEMPERATURE

If in the heat equation we taken into account the variations in the thermal properties with temperature, we must write:

(1) $$a(P)\frac{\partial^2 P}{\partial x^2} = \frac{\partial P}{\partial t}$$

where P represents the thermal potential given by $\int_0^T \lambda dT$. The zero level corresponds to the initial temperature of the pickup which is assumed to be at ambient temperature.

An interference method can be applied to resolve equation 1. Let:

(2) $$a = a_0 + a_1$$

with $a_1 << a_0$. In this formula a_0 represents the diffusivity at the initial temperature and a_1 the portion which varies with P. Likewise, we write:

(3) $P = P_0 + P_1$

where P_0 is the solution to an equation such as 1 corresponding to a_0. Hence the system:

(4) $$\begin{cases} a_0 \frac{\partial^2 P_0}{\partial x^2} = \frac{\partial P_0}{\partial t} \\ (a_0 + a_1)\left(\frac{\partial^2 P_0}{\partial x^2} + \frac{\partial^2 P_1}{\partial x^2}\right) = \frac{\partial P_0}{\partial t} + \frac{\partial P_1}{\partial t} \end{cases}$$

The calculation will be performed assuming the pickup is subjected to a thermal flux of constant density φ_0. The first equation of 4 has the following solution:

(5)

$$P_0 = \varphi_0 \left[2\sqrt{\frac{a_0 t}{\pi}} e^{-\frac{x^2}{4a_0 t}} - x \cdot \operatorname{erfc} \sqrt{\frac{x^2}{4a_0 t}} \right].$$

We can write the second equation of 4 while ignoring the term $a_1 \frac{\partial^2 P_1}{\partial x^2}$, as infinitely small of the second order and by using the value of $\frac{\partial^2 P_0}{\partial x^2}$ taken from the first equation of the system:

(6) $$a_0 \frac{\partial^2 P_1}{\partial x^2} = \frac{\partial P_1}{\partial t} - \frac{a_1}{a_0} \frac{\partial P_0}{\partial t}.$$

Let us assume that the temperature interval considered is such that the variations in the thermal properties follow more or less linear laws. We then have:

$$a_1 = \beta P_0.$$

The formulas are written by introducing a dimensionless space variable: $x^+ = \frac{x}{2\sqrt{a_0 t}}$. The portion P_1 of the thermal potential is of the form:

(7) $$P_1 = -\frac{8\beta\varphi_0^2 t}{\pi} F(x^+).$$

If we substitute the expression of a_1 in equation 6, that of P_0 taken from 5 and that of P_1 according to (7), we arrive at:

(8)
$$F'' + 2x^+F' - 4F = \\ = e^{-2x^{+2}} - \sqrt{\pi} \cdot x^+ \cdot e^{-x^{+2}} \cdot \operatorname{erfc} x^+.$$

The solution to equation 8 is of the form:

(9)
$$F(x^+) = A(2x^{+2} + 1) + Bx^+ \cdot e^{-x^{+2}} + \\ + \frac{8B\sqrt{\pi} - \pi}{16}(2x^{+2} + 1) \operatorname{erf} x^+ \\ + \frac{\sqrt{\pi}}{8} x^+ e^{-x^{+2}} \operatorname{erf} x^+ + \frac{\pi}{16}(2x^{+2} + 1) \operatorname{erf}^2 x^+.$$

The limit conditions are the following:

- For infinite x, T = 0, i.e., P = P_1 = 0 for infinite x^+.

This gives: $F(\infty) = 0$, hence:

$$A = -B\frac{\sqrt{\pi}}{2}$$

- For x = 0, the flux density is

$$\varphi_0 = -\frac{\partial P}{\partial x} = -\frac{\partial P_0}{\partial x} - \frac{\partial P_1}{\partial x}$$

19

But the solution P_0 was obtained by letting: $\varphi_0 = \frac{\partial P_0}{\partial x}$ for x = 0. It follows that: $\frac{\partial P_1}{\partial x}_{x=0} = 0$ at the surface, hence F'(0) = 0. From this we find: $B = \frac{\sqrt{\pi}}{16}$.

The function F therefore takes on the following value at the surface:

$$F(x' = 0) = A = -\frac{\pi}{32}$$

According to (7) the surface value of P_1 is written as:

(10) $$P_{1s} = \frac{\beta\varphi_0^2 l}{4}$$

For the portion P_0 we obtain from equation 5:

(11) $$P_{0s} = 2\varphi_0\sqrt{\frac{a_0 l}{\pi}}$$

The surface potential takes on the form:

(12) $$P_s = P_{0s} + P_{1s} = 2\varphi_0\sqrt{\frac{a_0 l}{\pi}} + \frac{\beta\varphi_0^2 l}{4}$$

In order to find the expression for the temperature we use the following equation for the law of variation of the thermal conductivity: $\lambda = \lambda_0 + \gamma T$. We then deduce the expression of the potential as a function of the surface temperature rise T_s:

(13) $$P_s = \lambda_0 T_s + \gamma\frac{T_s^2}{2}$$

By substituting into 13 the value (12) of P_s, we arrive at:

(14)

$$T_s = \frac{\lambda_0}{\gamma}\left(\sqrt{1+\frac{2\gamma\varphi_0\sqrt{t}}{\lambda_0^2}\left(2\sqrt{\frac{a_0}{\pi}}+\frac{3\varphi_0\sqrt{t}}{4}\right)}-1\right).$$

Letting: $T_s(0)=\frac{2\varphi_0\sqrt{t}}{\sqrt{\pi\lambda_0\rho_0c_0}}$, which represents the surface heating which would be obtained if the thermal properties remained constant; the development of (14) up to the second degree term provides the following equation:

(15)

$$T_s = T_s(0)\left[1+T_s(0)\left(\beta\cdot\frac{\pi\rho_0c_0}{16}-\frac{\gamma}{2\lambda_0}-\right.\right.$$
$$\left.\left.-\gamma\beta^2\frac{\pi^2\rho_0^2c_0^2}{512\lambda_0}T_s(0)^2-\gamma\beta\frac{\pi\rho_0c_0}{16\lambda_0}T_s(0)\right)\right].$$

Noting that:

$$\gamma T_s \ll \lambda_0 \text{ et } \beta P_0 \ll a_0, \text{ soit } \beta T_s \ll \frac{1}{\rho_0c_0},$$

equation 15 is more or less equal to:

(16)
$$T_s = T_s(0)\left[1+T_s(0)\left(\frac{3\pi\rho_0c_0}{16}-\frac{\gamma}{2\lambda_0}\right)\right].$$

If we base ourselves on the data in reference [3] between 20 and 50 degrees C for silica, we have: λ_0 = 14.5 mW/cm-degree-C; γ = 0.0067 mW/cm(degreesC)^2; ρ_0C_0 = 1.60 J/cm^3-degrees-C; β = -1.04 x 10^{-3}cm^3/J.

By substituting these values into 16:

$$T_s = T_s(0)\left[1-5{,}57\cdot10^{-4}T_s(0)\right].$$

If the surface temperature rise is 30 degrees C, the corrective term is equal to 9.983.

APPENDIX 4

LATERAL DIFFUSION OF HEAT IN ELECTRICAL CALIBRATION

The passing of a current in the thin layer only heats the calorimetric element from the surface located beneath this layer. Deviations occur with respect to a single-dimensional diffusion as soon as the thermal signal reaches a substantial depth compared to the dimensions of the thin layer. We shall assume that the layer is in the form of a rectangle of infinite length and width 1; in addition, we shall assume that the thin layer is perfectly homogeneous so that the thermal flux density φ_0, which is independent of time, is identical at all points in the heating element.

Let us first consider a punctual heat source which instantly frees a quantity of heat q within an infinite medium, for t = 0. At a point located at a distance r from the source, the Laplace transform θ of the temperature rise is the solution to the equation:

(1) $$a\left(\frac{\partial^2\theta}{\partial r^2}+\frac{2}{r}\frac{\partial\theta}{\partial r}\right)=p\theta$$

whree a represents the thermal diffusivity of the medium. The solution which takes into account θ = 0 for infinite r, and because all of the heat released is equal to q, is written:

(2) $$\theta=\frac{q}{4\pi\lambda r}e^{-\sqrt{\frac{p}{a}}r}$$

We deduce the temperature rise at r by inversion:

(3) $$T=\frac{q}{8\rho c\,(\pi at)^{3/2}}\cdot e^{-\frac{r^2}{4at}}.$$

The ratio $\frac{q}{\rho c}$ is called intensity of the heat source. If we now consider a source line of infinite length, running along the y-axis, which releases a quantity of heat q_1 per unit length at time t = 0, each element delivers q_1 dy and the heating

contribution at a point located at a distance r from the source line is deduced from equation 3:

(4)

$$dT = \frac{q_1 dy}{8\rho c\,(\pi a t)^{3/2}}\, e^{-\frac{r^2+y^2}{4at}}.$$

Hence:

(5)

$$T = \int_{-\infty}^{+\infty} \frac{\partial T}{\partial y}\, dy = \frac{q_1}{4\pi\lambda t}\, e^{-\frac{r^2}{4at}}.$$

If the source line releases a heat flux q_1 per unit length continuously starting with the initial time, the quantity of elementary heat delivered at instant τ is equal to $q_1' d\tau$; the contribution to the heat rise

at time t at a point located at a distance r from the line according to equation 5 is given by:

(6)

$$dT = \frac{q_1' d\tau}{4\pi\lambda\,(t-\tau)}\, e^{-\frac{r^2}{4a(t-\tau)}}$$

By integrating we obtain the heating at time t:

(7)

$$T = \int_0^t \frac{\partial T}{\partial \tau}\, d\tau = -\frac{q_1'}{4\pi\lambda}\, E_i\left(-\frac{r^2}{4at}\right)$$

where E_i designates the integral exponential function.

Up until now we have assumed that the source released heat in an infinite medium. If this source is located at the surface of a semi-unlimited material, then the temperature rise given by equation 7 must be doubled. From this modified equation, we can deal with the case of a rectangular surface strip of infinite length and width 1, the source of a thermal flux of constant density φ_0. The reference trihedral includes a y-axis f symmetry for the emitting strip, an x-axis located on the surface of the calorimetric medium, and a z-axis perpendicular to this

surface. The distance of a point in the medium having coordinates (z, x) with an elementary strip of abscissa x' and width dx' is expressed by:

(8) $$r = \sqrt{z^2 + (x - x')^2}.$$

This elementary strip emits per unit length a flux $\varphi_0 dx'$ and its contribution to the heating of the point considered is written as follows, based on equations 7 and 8, after having multiplied the second member of 7 by 2:

(9) $$dT = -\frac{\varphi_0 dx'}{2\pi\lambda} E_i\left[-\frac{(z^2 + [x - x']^2)}{4at}\right].$$

The total temperature rise is obtained by integration over the entire length of the emitting strip $(-l/2 \leqslant x' \leqslant +l/2)$; we are interested in the surface value T_S (for z = 0).

(10)

$$T_s = -\frac{\varphi_0}{2\pi\lambda}\int_{-l/2}^{+l/2} E_i\left[-\frac{(x - x')^2}{4at}\right] dx' =$$

$$= \frac{\varphi_0\sqrt{t}}{\sqrt{\pi\lambda\rho c}}\left[\operatorname{erf}\left(\frac{l/2 + x}{\sqrt{4at}}\right) + \operatorname{erf}\left(\frac{l/2 - x}{\sqrt{4at}}\right) - \right.$$

$$\left. \frac{1}{\sqrt{\pi}}\left(\frac{l/2 + x}{\sqrt{4at}} E_i\left[-\frac{(l/2 + x)^2}{4at}\right] + \frac{l/2 - x}{\sqrt{4at}} E_i\left(-\frac{(l/2 - x)^2}{4at}\right]\right)\right].$$

From equation 10 we calculate the average temperature rise on the surface of the material covered with the heat source:

(11) $$T_{sm} = \frac{l}{2}\int_0^{+l/2} T_s dx = \frac{2\varphi_0\sqrt{t}}{\sqrt{\pi\lambda\rho c}}\left[\operatorname{erf}\left(\frac{l}{\sqrt{4at}}\right) + \right.$$

$$\left. + \frac{\sqrt{4at}}{\sqrt{\pi}l}\left(e^{-\frac{l^2}{4at}} - 1 - \frac{l^2}{4at} E_i\left[-\frac{l^2}{4at}\right]\right)\right].$$

The factor term of the second member is the temperature rise that would be observed if the lateral were not taken into account. The term between the brackets depends on the parameter: $\beta = \frac{2\sqrt{at}}{l}$ which is equal--to within a numerical factor--to the ratio of the penetration depth of the signal to the width of the thin layer. In order to take into account the lateral diffusion, we must therefore multiply the measured surface heating values by the corrective factor:

$$\frac{1}{\operatorname{erf}\left(\frac{1}{\beta}\right) + \frac{\beta}{2\sqrt{\pi}}\left[e^{-\frac{1}{\beta^2}} - 1 - \frac{1}{\beta^2}\,E_i\left(-\frac{1}{\beta^2}\right)\right]}.$$

In an electrical calibration the value of β remains quite small. Let us consider, for example, a thin layer 0.5 mm wide deposited on silica and subjected to a test of 500 micro-sec; this corresponds to: $\beta \# 0.09$

For small values of β , we can use the following developments:

$$\begin{cases} \operatorname{erf}\dfrac{1}{\beta} = 1 - \dfrac{1}{\sqrt{\pi}}\,e^{-1/\beta^2}\left(\beta - \dfrac{\beta^3}{2} + \dfrac{3\,\beta^5}{4}\cdots\right) \\ E_i\left(-\dfrac{1}{\beta^2}\right) = e^{-1/\beta^2}\,(-\beta^2 + \beta^4\cdots). \end{cases}$$

Substituting into 11:

(12) $$T_{xM} = \frac{2\varphi_0\sqrt{t}}{\sqrt{\pi\lambda\rho c}}\left[1 - \frac{\beta}{2\sqrt{\pi}} + e^{-1/\beta^2}\left(\frac{\beta^5}{4\sqrt{\pi}} + \cdots\right)\right].$$

The term in e^{-1/β^2} is negligible for $\beta < 0.8$. We are therefore left with:

(13) $$T_{xM} = \frac{2\varphi_0\sqrt{t}}{\sqrt{\pi\lambda\rho c}}\,(1 - 0{,}282\,\beta).$$

For $\beta = 0.09$ the second member is equal to 0.975, that is, a correction of 2.5%.

APPENDIX 5

ELECTRICAL CALIBRATION ERROR CONNECTED WITH IRREGULARITIES IN THIN LAYERS

Let us take a thin layer of rectangular shape in which a known electrical power is dissipated to conduct a calibration of the fluxmeter. We can theoretically show that if a random irregularity (variation in thickness of width, structural heterogeneity) exists in this layer, causing local variations in the thermal flux density delivered, the results of the calibration are fraught with an error that may be substantial. The calculated value of the coefficient $\sqrt{\lambda \rho c}$ is always less than the true value.

The irregularities in the thin layers are generally random. As an example, we shall only give the calculation corresponding to a simple illustrative case, that is, a thickness variation in the widtn direction.

The value of the thickness of the thermometer element will be represented by z(x), the length and width by L and l respectively. The resistance R has the value R_o at the beginning of the test and the electrical conductivity is expressed by:

(1) $$\sigma = \sigma_0 (1 - \alpha T)$$

where α is the temperature coefficient. For simplification, we shall assume that σ and α are identical at all points of the thin layer, which represents an optimistic hypothesis, with the possible variations in these values necessarily causing an increase in the error made.

The rectangular strip can be considered a group of resistors in parallel supplied by a potential difference V. For an elementary strip of width dx, we have:

(2) $$d\left(\frac{1}{R_0}\right) = \frac{\sigma_0 z\,dx}{L}.$$

The density of the local heat flux emitted by this elementary strip is expressed by:

(3) $$\varphi(x) = \frac{V^2}{L\,dx}\,d\left(\frac{1}{R_0}\right) = \frac{V^2\sigma_0 z(x)}{L^2}.$$

We shall only consider the first instants of heating before the lateral diffusion effect takes on any importance. In this case each point on the surface of the calorimetric element is only heated by the portion of the thin layer covering it. The local surface temperature rise is therefore equal to:

(4) $$T_s(x) = \frac{2\,\varphi(x)\,\sqrt{t}}{\sqrt{\pi\lambda\rho c}} = \frac{2\,\sqrt{t}}{\sqrt{\pi\lambda\rho c}} \cdot \frac{V^2\sigma_0 z(x)}{L^2}.$$

The average heating of the surface located under the thin layer is expressed by:

(5) $$T_{sM} = \frac{1}{l}\int_l T_s(x)\,dx = \frac{2\,\sqrt{t}}{\sqrt{\pi\lambda\rho c}}\,\frac{V^2\sigma_0}{L^2 l}\int_l z(x)\,dx$$

with the integral extending over the entire width.

The surface temperature rise is thus proportional to the average flux density and follows a normal $\sqrt{t}$ law. Still, T_{sM} is not accessible to measurement and the error comes from the fact that the value determined experimentally is equal to $\frac{\Delta R}{R_0 z}$, if ΔR is the resistance variable.

From equation 2 we obtain:

(6) $$\frac{1}{R_0} = \frac{\sigma_0}{L}\int_l z(x)\cdot dx.$$

Taking into account the variation of σ with local temperature T_S (x) given by equation 4:

(7) $$\frac{1}{R_0 + \Delta R} = \frac{\sigma_0}{L} \int_l (1 - \alpha T_s)\, z\, dx.$$

By combining 6 and 7 and noticing that the term $\alpha . T_S$ is small compared to unity, we arrive more or less at:

(8) $$\frac{\Delta R}{R_0 \alpha} = \frac{\int_l T_s \cdot z\, dx}{\int_l z\, dx}.$$

By substituting the value of T_S taken from equation 4:

(9) $$\frac{\Delta R}{R_0 \alpha} = \frac{2\sqrt{t}}{\sqrt{\pi \lambda \rho c}} \cdot \frac{V^2 \sigma_0}{L^2} \cdot \frac{\int_l z^2 dx}{\int_l z\, dx}.$$

From equation 5 we obtain:

(10) $$\sqrt{\lambda \rho c} = \frac{2\sqrt{t}}{\sqrt{\pi}\, T_{sM}} \frac{V^2 \sigma_0}{L^2 l} \int_l z\, dx.$$

If, instead of T_{SM} we introduce the value deduced for the first member of equation 10 in the formula $\frac{\Delta R}{R_0 \alpha}$, given by equation 9, that is, the calculated $(\sqrt{\lambda \rho c})$ is expressed by:

(11) $$(\sqrt{\lambda \rho c})_{calculé} = \sqrt{\lambda \rho c} \cdot \frac{\left(\int_l z\, dx\right)^2}{l \int_l z^2\, dx}.$$

Let us assume that the profile of a section of the layer takes on a more or less parabolic symmetrical form as suggested in reference [14] according to measurements made on a deposit obtained by platinum painting. We have:

$$z(x) = \varepsilon \left(1 - \frac{4x^2}{l^2}\right).$$

If we insert this expression in equation 11, we get: 22

$$(\sqrt{\lambda\rho c})_{\text{calculé}} = \frac{5}{6}\sqrt{\lambda\rho c}.$$

We demonstrate from Schwartz' inequality that the expression $\frac{\left(\int_1 z dx\right)^2}{l\int_1 z^2 dx}$ is always less than unity if z is variable. This ratio is only equal to 1 when z is constant.

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